Math 6610
Fall 2016

Homework 1, Due September 9 2016.
Show all the work. Late homework will not be accepted.

Problem 1.
Trefethen, page 15, problem 2.1
Problem 2.
a) Trefethen, page 15, problem 2.3
b) Trefethen, page 16, problem 2.4
Problem 3.
Trefethen, page 16, problem 2.6
Problem 4.
Trefethen page 24, problem 3.2
Problem 5.
a)Let $N(x) := ||\cdot||$ be any vector norm on C^{n} (or R^{n}). Show that $N(x)$ is a continuous function of the components $x_1, x_2, ..., x_n$ of x.
b) Prove that if $W \in \mathrm{C}^{\mathrm{m}\times\mathrm{m}}$ is an arbitrary nonsingular matrix, and $||\cdot||$ is any norm on C^{m}, then $||x||_W = ||Wx||$ is a norm on C^{m}.
c) Explain why $||I|| = 1$ for every induced matrix norm.
d) What is $||I_{n\times n}||_F$?
e) Show that Frobenius norm is not induced by any vector norm.
f) (Young's inequality) Let $p, q > 1, \frac{1}{p} + \frac{1}{q} = 1$. Then, show that for any two nonnegative real numbers a and b,

$$ab \leq \frac{a^p}{p} + \frac{b^q}{q}$$

g) (Hölder's inequality) $p, q > 1, \frac{1}{p} + \frac{1}{q} = 1$. Then, show that for any $u \in \mathrm{R}^{\mathrm{n}}$ and $v \in \mathrm{R}^{\mathrm{n}}$, we have

$$|\sum_{i=1}^{n} u_i v_i| \leq ||u||_p ||v||_q.$$

h) Show that for an $n \times n$ matrix $A = (a_{ij})_{1\leq i,j\leq n} \in \mathrm{R}^{\mathrm{n}\times\mathrm{n}}$,

$$||A||_\infty = \max_{1\leq i\leq n} \sum_{j=1}^{n} |a_{ij}|$$