Name:

## Student ID #:

Practice Final Exam.

7 problems are given. Show all the work and Justify your steps. No notes, books, calculators are allowed.

| Problem | Points       | Score |
|---------|--------------|-------|
| 1       | 25           |       |
| 2       | 25           |       |
| 3       | 25           |       |
| 4       | 25           |       |
| 5       | 25           |       |
| 6       | 25           |       |
| 7       | 25           |       |
|         | Total Score: |       |

Problem 1.

Solve the differential equations: a)

$$y' = y^2 t.$$

b)

$$y(\sqrt{1+y^2})y' = xe^x.$$

c) 1. Find the solution of the differential equation that satisfies the given initial condition:

$$\frac{dP}{dt} = \sqrt{Pt}, \quad P(1) = 2$$

2. Assume that the above differential equation is a model for the position of an object at time t, P(t) is a position variable at time t. Find a position P(t) at time t = 9.

Problem 2.

Test the series for convergence or divergence: a)

 $1 + \frac{1}{8} + \frac{1}{27} + \frac{1}{64} + \frac{1}{125} + \dots$ 

b)

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{3n^2 + 1}$$

c)

$$\sum_{n=1}^{\infty} (-1)^n \frac{3n^2 - 1}{n^2 + 1}$$

d)

$$\sum_{n=1}^{\infty} \frac{n}{2n^3 + 1}$$

Problem 3.

Find the radius of convergence and interval of convergence of the series. a)  $\sum_{n=1}^{\infty} (-1)^n n^n$ 

b)  
$$\sum_{n=0}^{\infty} \frac{(-1)^n x^n}{2n+1}$$

$$\sum_{n=0}^{\infty} \frac{(x-3)^n}{n^5+1}$$

c)

$$\sum_{n=1}^{\infty} \frac{3^n (x+4)^n}{\sqrt{n}}$$

Problem 4.

a) Find the Taylor polynomial  $T_n(x)$  of degree n for the function f(x) around x = a.

b) Use Taylor's Inequality to estimate the accuracy of the approximation  $f(x) \approx T_n(x)$  when x lies in the given interval.

1.

 $f(x) = \sqrt{x}, \quad a = 1, \quad n = 3, \quad 0.9 \le x \le 1.1$ 

2.

$$f(x) = x^{2/3}, \quad a = 1, \quad n = 3, \quad 0.7 \le x \le 1.3$$

Problem 5.

a) Find the equation of the plane passing through the points A(0, 1, 1), B(1, 0, 1), and C(1, 1, 0).

b) The plane through the origin and the points (2, -4, 6) and (5, 1, 3)

c) The plane through the point (6,3,2) and perpendicular to the vector <-2,1,5>

d) Write a vector and parametric equation of the line passing through the point (6, -5, 2) and parallel to the vector  $\boldsymbol{v} = \langle 1, 3, -\frac{2}{3} \rangle$ .

e) Find parametric equations for the line segment from (10, 3, 1) to (5, 6, -3).

Problem 6.

a) Show that if a particle moves with constant speed, then the velocity and acceleration vectors are orthogonal.

b) A ball is thrown at an angle of  $\frac{\pi}{4}$  to the ground. If the ball lands 100m away, what was the initial speed of the ball?

Problem 7.

a) Find the limit, if it exists or show that the limit does not exist:
1.

1.  

$$\lim_{(x,y)\to(0,0)} \frac{x^3 - y^3}{2x^3 + y^3}$$
2.  

$$\lim_{(x,y)\to(0,0)} \frac{x^2 + \sin^2 y}{2x^2 + y^2}$$
3.  

$$\lim_{(x,y)\to(1,0)} e^{-x-y} \sin(5x + y^2)$$

b) Consider the function  $f(x, y) = x^3 - 7x^2 + y^2 + 1$ . Find all first and second order derivatives of f(x, y).

c) Consider the function  $f(x, y) = \cos(x - y)$ . Find all first and second order derivatives of f(x, y).