1 **The van Hiele Levels**

In 1957, Dutch educators Dina van Hiele-Geldof and Pierre van Hiele proposed that the development of a student's understanding of reasoning and proof progresses through five distinct levels.

2 **Level 0: Visual**

*Students identify and reason about shapes and other geometric configurations based on shapes as visual wholes rather than on geometry properties.
* For instance, they might identify a rectangle as a "door shape“ object.
*They would identify two shapes as congruent because they look the same, not because of shared properties.
*Students see a square and a “diamond” as different shapes, i.e. appearance defines shapes

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3 **Level 1: Descriptive/Analytic**

* Students recognize and characterize shapes by their properties.
- For example, they can identify a rectangle as a shape with opposite sides parallel and four right angles.
- Students at this level still do not see relationships between classes of shapes (e.g., all rectangles are parallelograms), and they tend to name all properties they know to describe a class, instead of a sufficient set.
- They cannot generalize attributes—how/why objects fit into classes beyond external reliance on definitions

4 **Level 2: Abstract/Relational (Informal and Deduction**

* Students are able to form abstract definitions and distinguish between necessary and sufficient sets of conditions for a class of shapes, recognizing that some properties imply others.
- Students also first establish a network of logical properties and begin to engage in deductive reasoning, though more for organizing than for proving theorems.
- Begin to use “if-then” reasoning
- Form logical arguments using properties
- Proofs are informal and intuitive

5 **Level 3: Formal Deduction and Proof (Deduction)**

*Students are able to prove theorems formally within a deductive system.
- They are able to understand the roles of postulates, definitions, and proofs in geometry, and they can make conjectures and try to verify them deductively.
- Informal arguments move to formal proofs
- Students can identify a minimal set of assumptions
- Students can identify what appears to be true by intuition vs. what is true by proof/thm/axiom

6 **Level 4: Rigor**

- Studentss operate at this highest level.
- Axiomatic systems are object of study
- Students develop an appreciation of different axiomatic systems
- Students can compare and contrast different axiomatic systems

7 **Teaching**

According to the van Hieles students must progress through *each* level. Developing each level is a result of instruction that is organized into five phases of instruction for learning

8 ***Information:***

The teacher identifies what students already know about a topic and the students become oriented to the new topic.

*how do teachers identify what a student already knows?

*how does the student become “oriented to the new topic”?

*What does “oriented” mean?

9 ***Guided orientation:***

Students explore the objects of instruction in carefully structured tasks such as measuring, folding, or constructing. The teacher ensures that students explore specific concepts.

*How does the teacher structure the task?

*How does the teacher ensure that specific concepts are explored?

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10 ***Explicitation:***

Students describe what they have learned about the topic. The teacher introduces relevant mathematical terms and help to formalize thinking.

*How can students describe what they have learned (with 40 kids in a class)

*How do teachers identify what are relevant terms?

*How do teachers determine what thinking to formalize?

11 ***Free Orientation:***

Students apply the relationships they are learning to solve problems and investigate more open-ended tasks.

*What is the difference between a task that identifies what a student already knows to one that moves a student to apply relationships to solve problems and/or investigate new concepts?

*How do you construct an open-ended task with a class of 40?

12 ***Integration:***

Students summarize and integrate what they have learned, developing a new network of objects and relations.

*How complicated do the open-ended tasks need to be?

*How are these levels of instruction associated with van Hiele levels of understanding?

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The van Hiele Levels of Geometric Understanding

Common Questions and Answers

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Q. Is the development of geometric understanding related to age or maturation? experience? instruction?

A . Progress from one level to the next level is more dependent on educational experiences than on age or maturation. Some experiences can facilitate (or impede) progress within a level or to a higher level.

15 **Q. Can a student skip levels?**

A . According to the van Hieles' model, a student cannot achieve one level of understanding without having mastered all the previous levels. Research in the United States and other countries supports this view with one exception. Some mathematically talented students appear to skip levels, perhaps because they develop logical reasoning skills in ways other than through geometry.

16 **Q. What if the teacher is thinking at a different van Hiele level than the students?**

A . This situation is common. Most high school geometry teachers think at the fourth or fifth van Hiele level. Research indicates that most students starting a high school geometry course think at the first or second level. The teacher needs to remember that although the teacher and the student may both use the same word, they may interpret it quite differently. For example, if a student is at the first level, the word "square" brings to mind a shape that looks like a square, but little else. At the second level, the student thinks in terms of the properties of a square, but may not know which ones are necessary or sufficient to determine a square. The student may feel that in order to prove that a figure is a square, all the properties must be proved. The teacher, who is thinking at a higher level, knows not only the properties of a square, but also which ones can be used to prove that a figure is a square. In fact, the teacher may think of several different ways to show that a figure is a square, since the teacher knows the relationships between the various properties and can determine which properties are implied by others. The teacher must evaluate how the student is interpreting a topic in order to communicate effectively.

17 **Q. What happens if a teacher tries to teach at a level of thought that is above a student's level?**

A . Generally, the student will not understand the content that is being taught. Usually, the student will try to memorize the material and may appear to have mastered it, but the student will not actually understand the material. Students may easily forget material that has been memorized, or be unable to apply it, especially in an unfamiliar situation.

18 **Q. What is the role of language in learning geometry?**

A . Language plays an important role in learning. As indicated above, each level of thought has its own language and its own interpretation of the same term. Discussing and verbalizing concepts are important aspects of the Information, Explicitation, and Integration phases of learning. Students clarify and reorganize their ideas through talking about them.

19 **Q. How can I assess a student's van Hiele level?**

A . There are tests that can be used to assign a van Hiele level. Within a classroom, however, it is more practical for a teacher to assess a student's van Hiele level by analyzing his or her responses to specific geometric tasks.

20 **Q. What are the implications of the van Hiele theory for my instructional practices?**

A . The van Hiele theory indicates that effective learning takes place when students actively experience the objects of study in appropriate contexts, and when they engage in discussion and reflection. According to the theory, using lecture and memorization as the main methods of instruction will not lead to effective learning. Teachers should provide their students with appropriate experiences and the opportunities to discuss them. Teachers can assess their students' levels of thought and provide instruction at those levels. The teacher should provide experiences organized according to the phases of learning to develop each successive level of understanding.