## Models of incidence geometry:

## Model \#1:

- Points: A, B, C
- Lines: $\{\mathrm{A}, \mathrm{B}\},\{\mathrm{A}, \mathrm{C}\},\{\mathrm{B}, \mathrm{C}\}$
- Point lies on $l$ if the letter belongs to the set $l$.


## Model \#2:

- Points: A, B, C, D
- Lines: $\{A, B\},\{A, C\},\{A, D\},\{B, C\},\{B, D\},\{C, D\}$
- Point lies on $l$ if the letter belongs to the set $l$.


## Model \#3 (Cartesian plane):

- Points: ordered pairs of real numbers (x, y)
- Lines: triples of real numbers ( $a, b, c$ ) so that either $a \neq 0$ or $b \neq 0$. It is the set of all points ( $\mathrm{x}, \mathrm{y}$ ) that satisfy the equation $\mathrm{ax}+\mathrm{by}+\mathrm{c}=0$.
- Point lies on $l$ if it is a solution of the $l$ 's equation.


## Model \#4 (Real projective plane):

- Points: unordered pairs $\{(\mathrm{x}, \mathrm{y}, \mathrm{z}),(-\mathrm{x},-\mathrm{y},-\mathrm{z})\}$, where $(\mathrm{x}, \mathrm{y}, \mathrm{z})$ lies on the unit sphere
- Lines are sets of points $\{(\mathrm{x}, \mathrm{y}, \mathrm{z}),(-\mathrm{x},-\mathrm{y},-\mathrm{z})\}$ that are parts of great circles on the unit sphere.
- Point lies on the line if both ( $\mathrm{x}, \mathrm{y}, \mathrm{z}$ ), ( $-\mathrm{x},-\mathrm{y},-\mathrm{z}$ ) lie on the corresponding great circle.


## Model \#5 (Hyperbolic plane):

- Points: ordered pairs of real numbers ( $\mathrm{x}, \mathrm{y}$ ), where $\mathrm{y}>0$.
- Lines:
o Subsets of vertical lines that consist of points ( $x, y$ ), with $y>0$
o Semicircles whose centers are points ( $\mathrm{x}, 0$ ), where x is a real number

Models of affine geometry (3 incidence geometry axioms + Euclidean PP) are called affine planes and examples are

Model \#2
Model \#3 (Cartesian plane).

## Model of (3 incidence axioms + hyperbolic PP) is <br> Model \#5 (Hyperbolic plane).

Models of projective geometry are called projective planes. Projective geometry consists of axioms $I-1, I-2+, I-3$ and Elliptic PP. $I-2+$ states: For every line $l$ there are at least three distinct points lying on it. Examples of projective planes are:

Model \#6 (projective completion of Model \#2):

- Points: A, B, C, D, E, F, G
- $\{A, B, E\},\{A, C, F\},\{A, D, G\},\{B, C, G\},\{B, D, F\},\{C, D, E\},\{E, F, G\}$
- Point lies on $l$ if the letter belongs to the set $l$.

Model \#4 Real projective plane (projective completion of Cartesian plane)

