## Models of incidence geometry:

### Model #1:

- Points: A, B, C
- Lines: {A, B}, {A, C}, {B, C}
- Point lies on *l* if the letter belongs to the set *l*.

## Model #2:

- Points: A, B, C, D
- Lines:  $\{A, B\}, \{A, C\}, \{A, D\}, \{B, C\}, \{B, D\}, \{C, D\}$
- Point lies on *l* if the letter belongs to the set *l*.

## Model #3 (Cartesian plane):

- Points: ordered pairs of real numbers (x, y)
- Lines: triples of real numbers (a, b, c) so that either  $a \neq 0$  or  $b \neq 0$ . It is the set of all points (x, y) that satisfy the equation ax + by + c = 0.
- Point lies on *l* if it is a solution of the *l*'s equation.

# Model #4 (Real projective plane):

- Points: unordered pairs {(x,y,z), (-x,-y,-z)}, where (x,y,z) lies on the unit sphere
- Lines are sets of points {(x,y,z), (-x,-y,-z)} that are parts of great circles on the unit sphere.
- Point lies on the line if both (x,y,z), (-x,-y,-z) lie on the corresponding great circle.

# Model #5 (Hyperbolic plane):

- Points: ordered pairs of real numbers (x, y), where y > 0.
- Lines:
  - Subsets of vertical lines that consist of points (x, y), with y > 0
  - Semicircles whose centers are points (x, 0), where x is a real number

**Models of affine geometry** (3 incidence geometry axioms + Euclidean PP) are called affine planes and examples are

Model #2 Model #3 (Cartesian plane).

Model of (3 incidence axioms + hyperbolic PP) is Model #5 (Hyperbolic plane). **Models of projective geometry** are called projective planes. Projective geometry consists of axioms *I*-1, *I*-2+, *I*-3 and Elliptic PP. *I*-2+ states: For every line *l* there are at least three distinct points lying on it. Examples of projective planes are:

**Model #6** (projective completion of Model #2):

- Points: A, B, C, D, E, F, G
- $\{A, B, E\}, \{A, C, F\}, \{A, D, G\}, \{B, C, G\}, \{B, D, F\}, \{C, D, E\}, \{E, F, G\}$
- Point lies on *l* if the letter belongs to the set *l*.

Model #4 Real projective plane (projective completion of Cartesian plane)