Math 431 Homework 3
Due 10/2

1. Show that the following familiar interpretation is a model for incidence geometry.

- The "points" are all ordered pairs (x, y) of real numbers.
- A "line" is specified by an ordered triple (a, b, c) of real numbers such that either $a \neq 0$ or $b \neq 0$; it is defined as the set of all "points" (x, y) that satisfy the equation $ax + by + c = 0$.
- "Incidence" is defined as set membership.

Recall that a statement in an axiomatic system is *independent* if it is not a theorem and its negation is not a theorem. To conclude that a statement S is independent, you need to find a model that satisfies S and a model that does not satisfy S. In problems 1, 2, and 3, you are given a statement S. For each problem, decide if (a) S is a theorem in incidence geometry; (b) not-S is a theorem in incidence geometry; or (c) S is an independent statement in incidence geometry. Provide justification (give a proof or supply the appropriate models). You may quote results that were proved in class or on homework. Remember that when giving a model, you should check that it satisfies the axioms.

2. There exist four distinct lines.

3. There exists a point such that at most one line passes through it.

4. For any three lines l, m, and n, if $l \parallel m$ and $n \nparallel l$, then $n \nparallel m$.

5. An *affine plane* is a model for incidence geometry having the Euclidean parallel property: For every line l and every point P not on l, there exists exactly one line through P that is parallel to l.

Prove that in an affine plane, for any three lines l, m, and n, if $l \parallel m$ and $n \nparallel l$, then $n \nparallel m$. (Thus, the statement given in problem 4 is a theorem in affine geometry.)