Class #8

Models, planes and such

Abstract axiomatic system ⟺ Model for the system

S is a theorem ⟹ S is correct

S can not be proved ⟸ S is not correct

S here is a statement in the abstract system

- ***I-1*** is not a correct statement in interpretation #2
  - ***I-1*** can not be proved from ***I-2*** and ***I-3.***
- ***I-1*** is a correct statement in interpretation #1
  - ***Not I-1*** can not be proved from ***I-2*** and ***I-3.***
- ***I-1*** is *independent* from ***I-2*** and ***I-3***

Independence

- Definition: A statement S in an axiomatic system is called independent if there is no proof of S and there is no proof of ~S.

- Q: How can you show that Euclid's Parallel Postulate is independent of IG?

Interpretation of IG #3

- Points – A, B, C, D
- Lines – {A,B}, {A,C}, {A,D},{B,C},{B,D},{C,D}

We checked that this is in fact a model for IG and that EPP is correct in this model.

In Model #1 (3 points and 3 lines, from last class), EPP is not correct. We conclude:

EPP is independent of IG

- Definition: An axiomatic system is said to be *complete* if there are no independent statements in the language of the system.

- Q: Is incidence geometry complete?
- A: No, because EPP is an independent statement.

Isomorphism of the models

- Definition: Two models of an axiomatic system are said to be isomorphic if there is a one-to-one correspondence between the basic objects that preserves the relationship between the objects.

- For IG: there is a one-to-one correspondence between points and lines so that if a point P lies on a line *l* in one model the point corresponding to P lies on the line corresponding to the line *l* in the other system.

Categorical systems

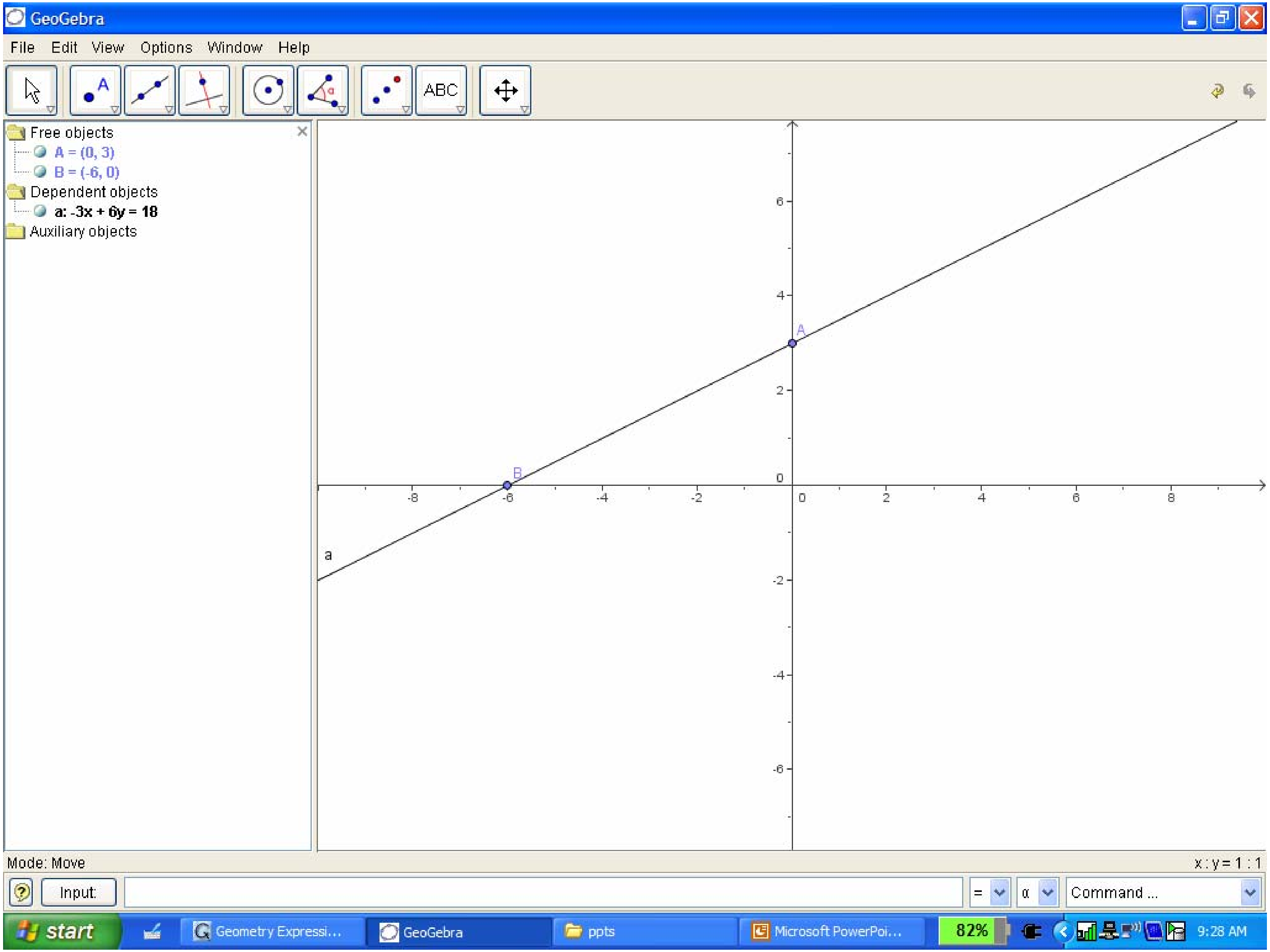
- If an axiomatic system has only one model (up to isomorphism) then it is called categorical.

- They completely describe all the properties of that model.

- Q: Can you think of a categorical system?
- Sarah Chow: If we change ***I-3*** to say: There are only three distinct points and no line passes through all three of them, the new system is categorical.

Cartesian plane

- Points are ordered pairs of real numbers (x, y)
- Lines are triples of real numbers (a, b, c) so that either $a \neq 0$ or $b \neq 0$. It is the set of all points (x, y) that satisfy the equation $ax + by + c = 0$.
- Lies on $\equiv$ is a member of


GeoGebra
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ABC
Free objects
A = (0, 3)
B = (-6, 0)
Dependent objects
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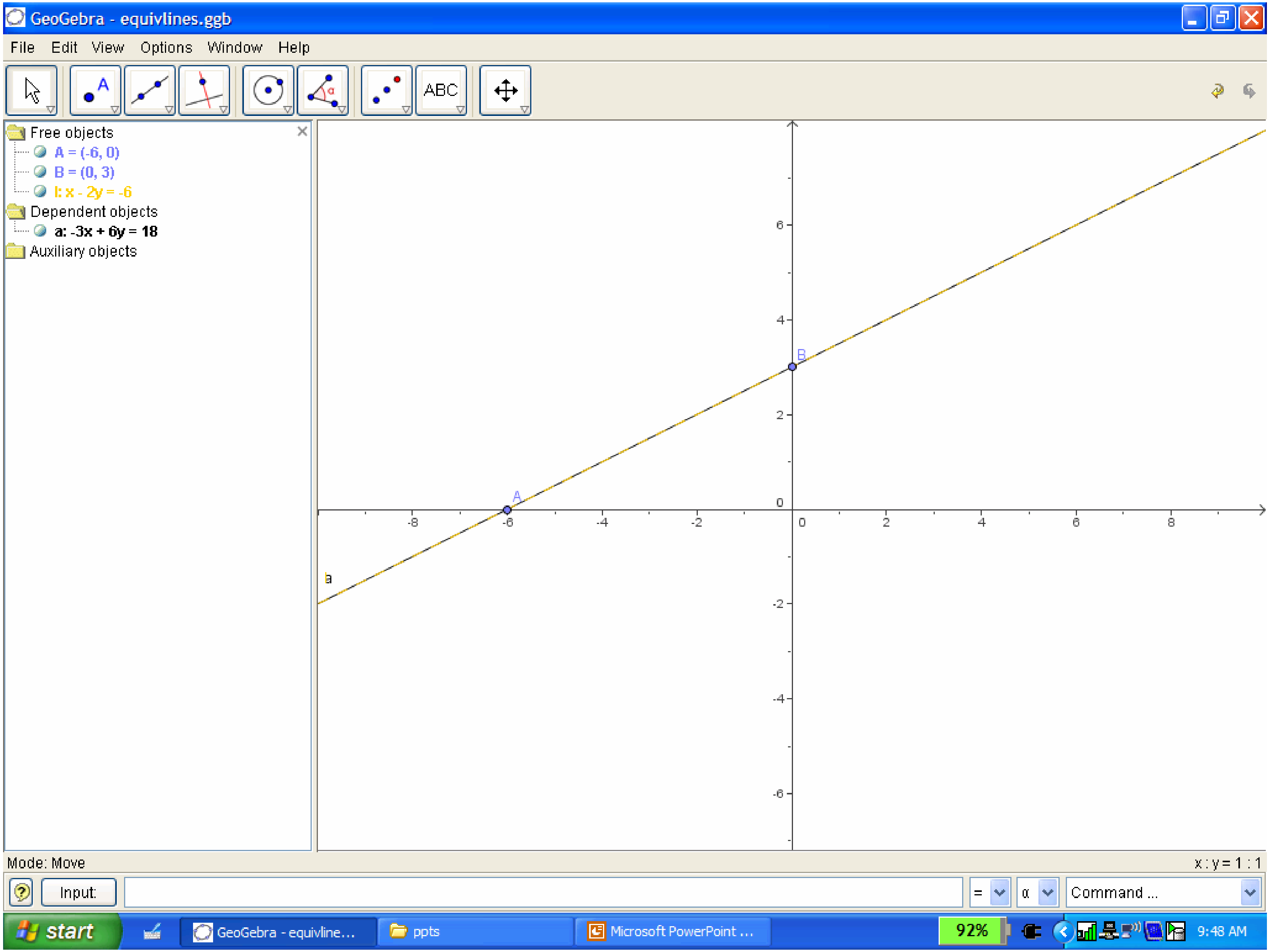
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a: -3x + 6y = 18
Auxiliary objects
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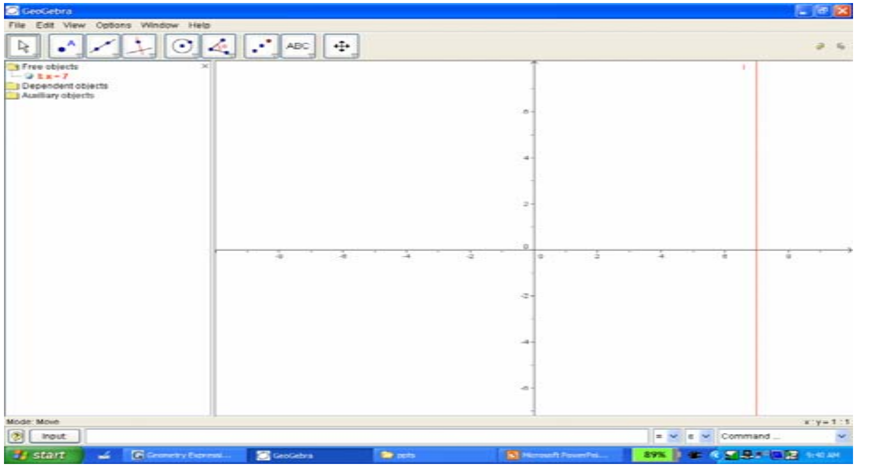
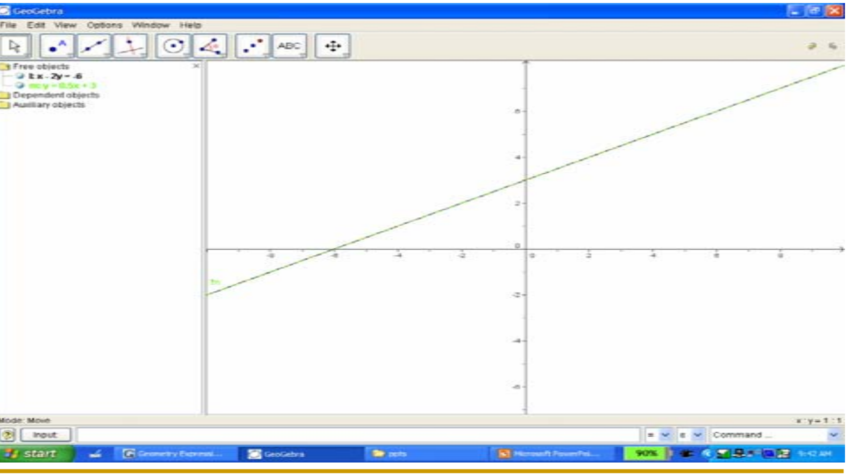
ax+by+c=0

- b=0: equation ax+by+c=0 is equivalent to one of the form x=constant.



- b≠0: equation ax+by+c=0 is equivalent to one of the form y=mx+n



Exercise:

- Show that the Cartesian plane is a model of incidence geometry.
- Show that the Cartesian plane satisfies the EPP (For every line *l* and every point P not lying on *l*, there is a unique line through P parallel to *l*).