Class #34

Sum of angles

Types of quadrilaterals

- You defined or talked about various types of quadrilaterals:
  - Square – all sides are congruent and all angles are right angles
  - Rectangle – all angles are right angles
  - Parallelograms – opposite sides are parallel
  - Kite – there are two distinct pairs of congruent adjacent sides
  - Rhombus – all sides are congruent

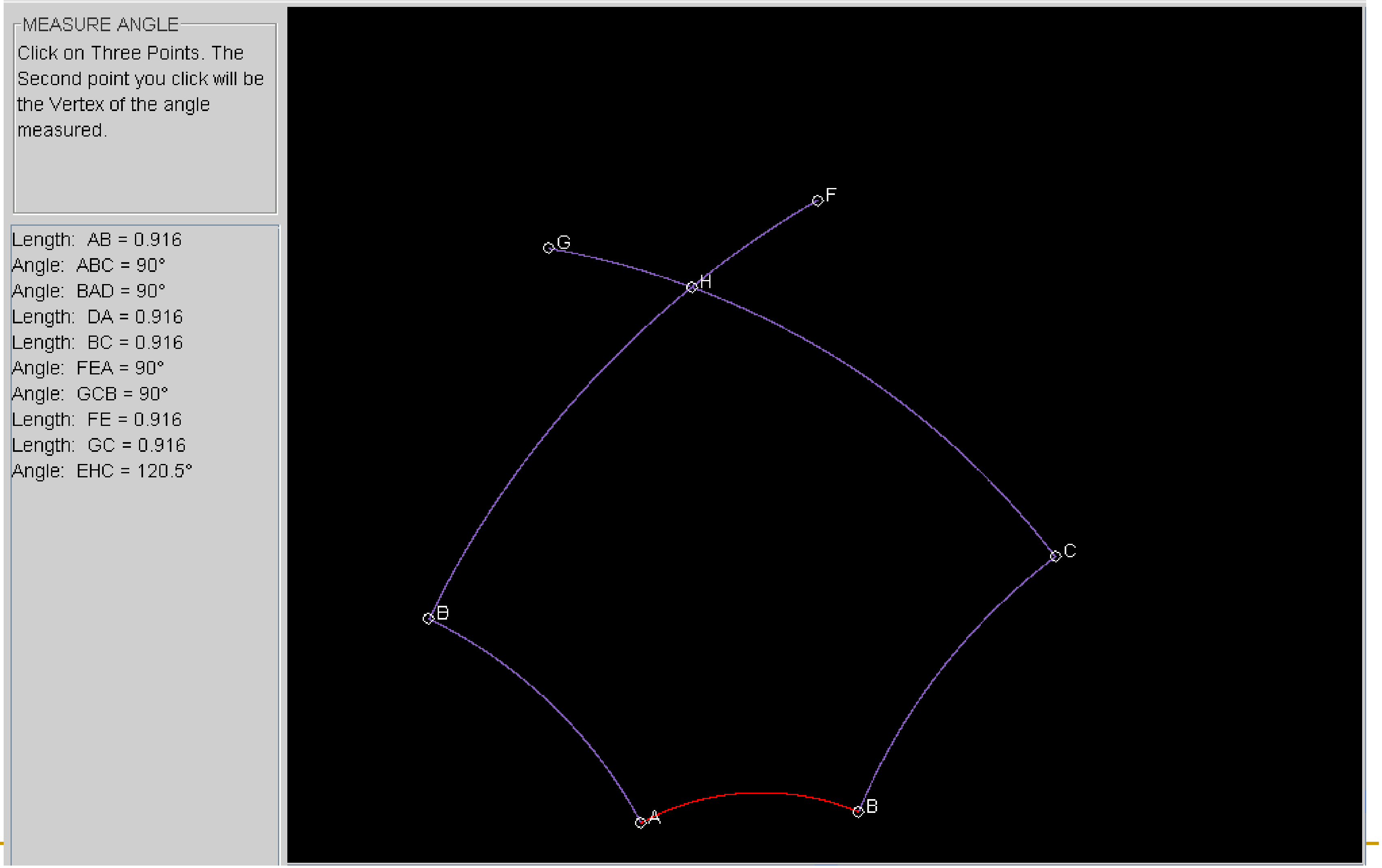
Questions to ask

- Do these exist?
- If they do, what are their relationships?
  - Every square is a rectangle?
  - Every rectangle is a parallelogram?
  - Every square is a rhombus?
  - Every rhombus is a kite?
  - If a quadrilateral has four congruent sides and four congruent angles then it is a square?

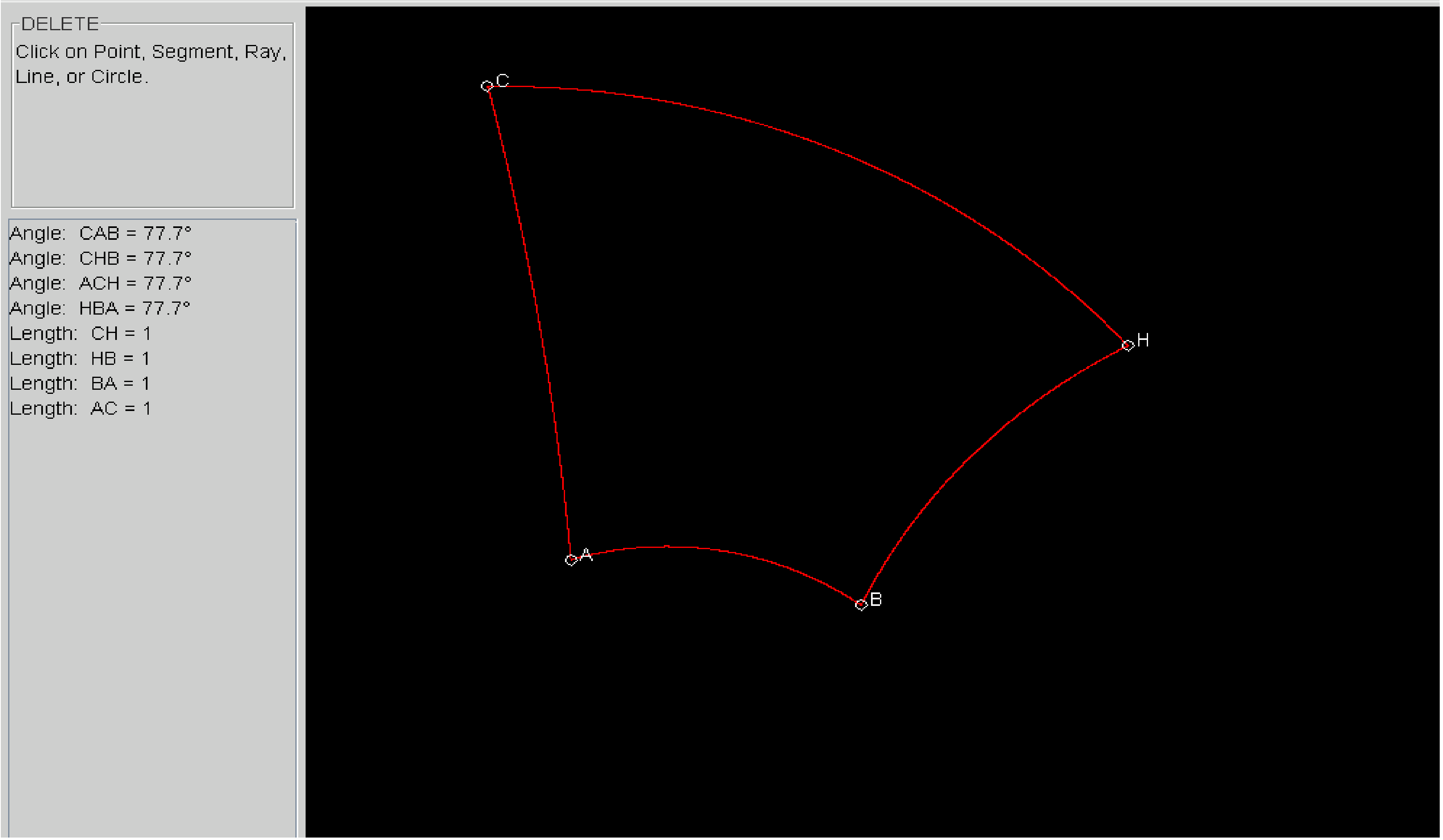
Rectangle

- Let's construct a rectangle in hyperbolic geometry:
  - NonEuclid

4 right angles and 4 congruent sides – forces fifth side in hyperbolic geometry



Quadrilateral with all sides congruent and all angles congruent.

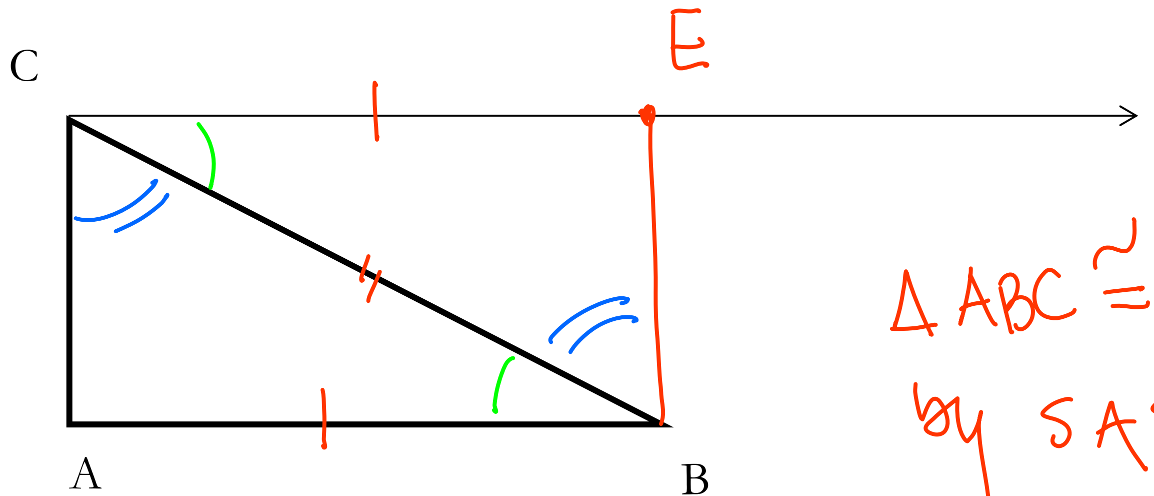


When does a rectangle exist?

- Rephrased:
  - When can you construct a rectangle?
- Would you be able to do it if you had a right angle triangle?
  - NonEuclid
- Is there something else you'd need to know about that triangle?
  - Would it help if the sum of the measures of the angles was 180°?

$m(\sphericalangle A)=\alpha$, $m(\sphericalangle B)=\beta$, $m(\sphericalangle C)=\gamma$

If there is a right triangle whose angle sum is 180°, then a rectangle exist.



$\alpha+\beta+\gamma= 180°$

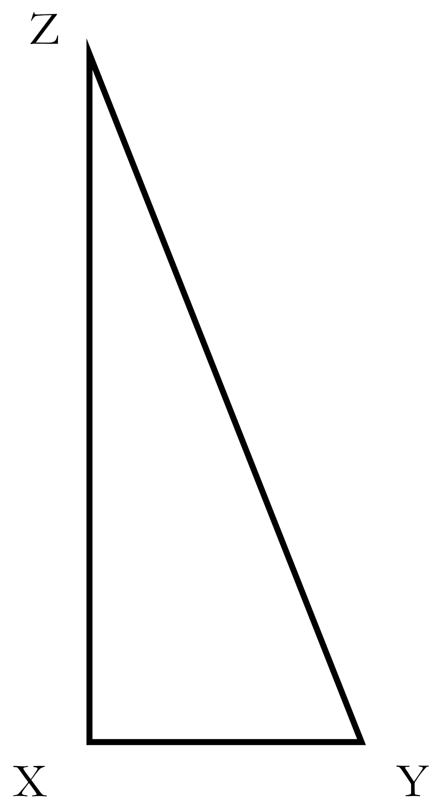
$\Delta ABC \cong \Delta ECB$ by SAS $\Rightarrow$

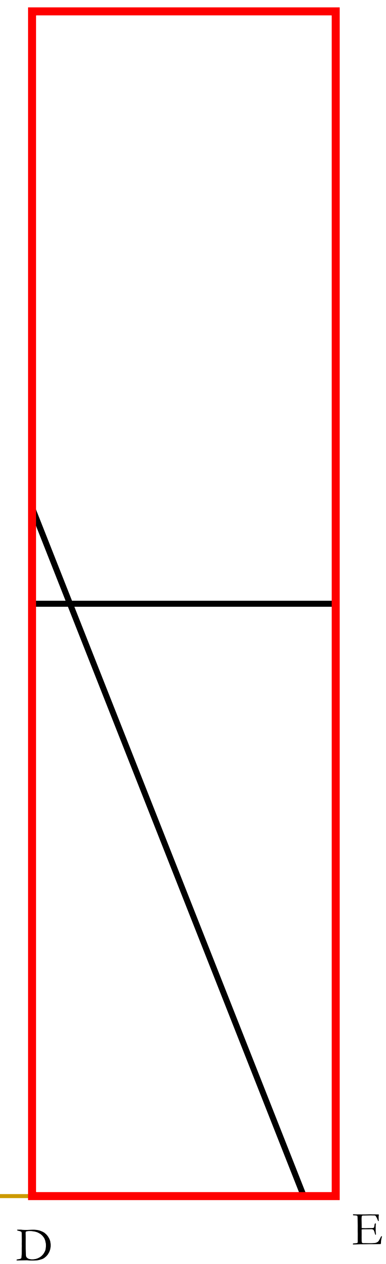
$\measuredangle EBC \cong \measuredangle ACB$

$m(\measuredangle ECA) = \beta+\gamma = 90°$

If there is a rectangle, then there are arbitrarily large rectangles.

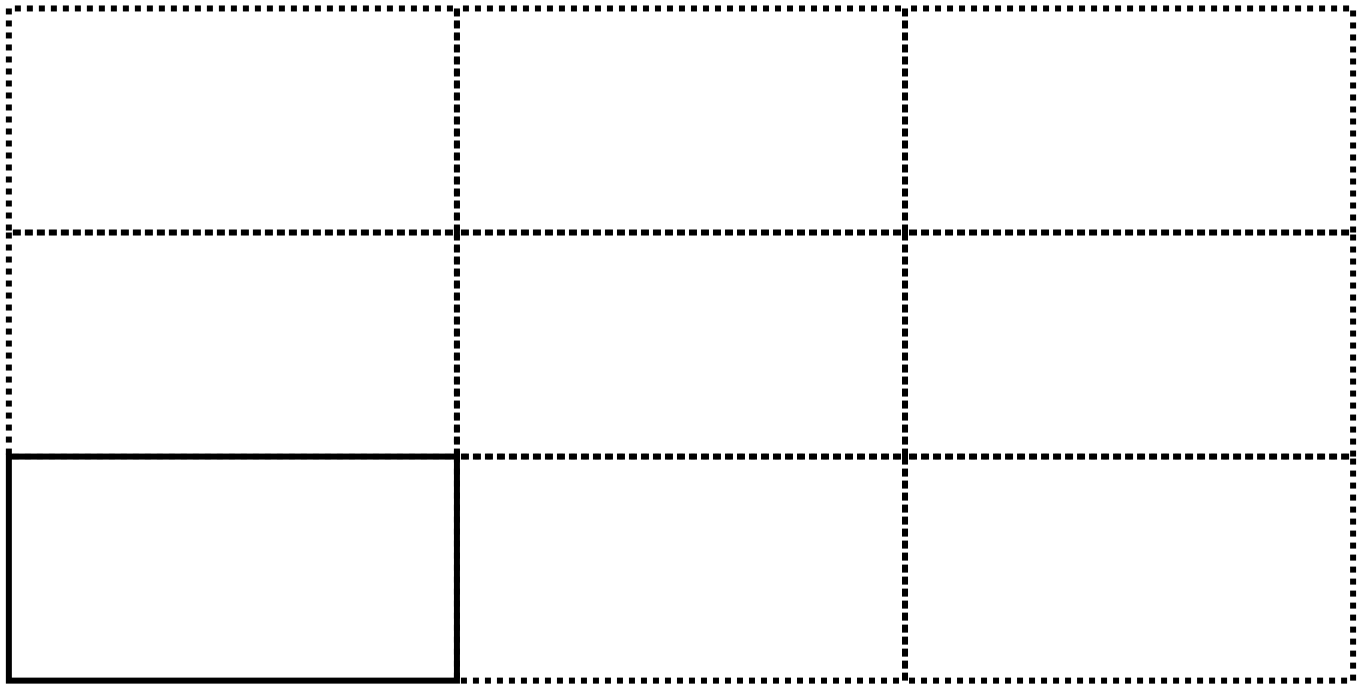
- Given a right triangle ΔXYZ (with right angle at X), then there is a rectangle □DEFG such that DE>XY and DG>XZ.





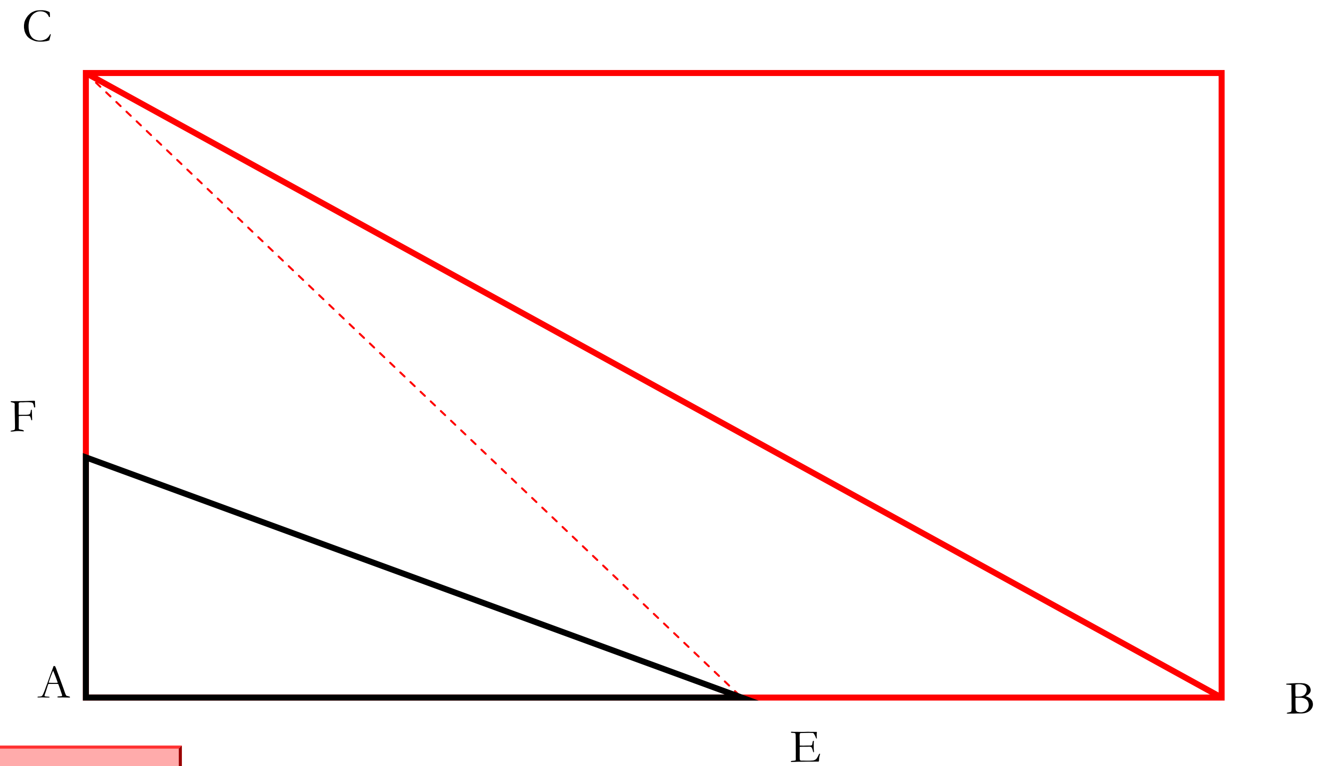
Book says:

To be slightly more rigorous you could argue that you can build large rectangles using the initial right triangle.

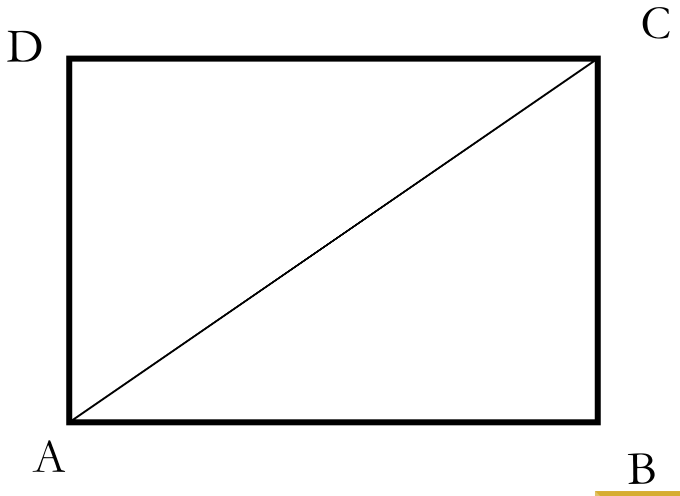


If one right angle triangle has angle sum 180º, then all right angle triangles have angle sum 180º.

If you know that ΔABC has angle sum 180º, could you show that
The same holds for ΔAEF?



- If the angle sum in □ABCD is 360°, then both ΔABC and ΔACD have angle sum 180°.



Suppose one of them had angle sum less than 180°. Then the other would have to Have angle sum greater than 180°, if they were to add up to 360°.

Can a triangle have angle sum greater than 180°?