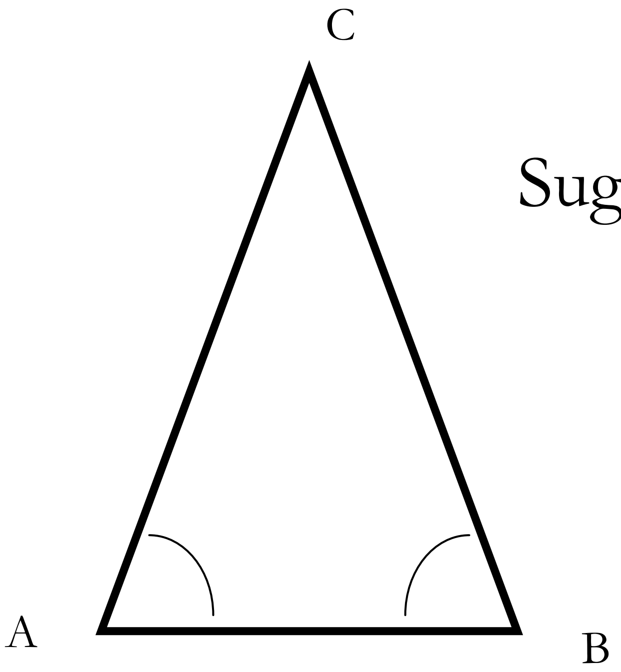
Class #25

Towards continuity principles and ultimately some familiar waters

Base angle theorem revisited.

If in a ΔABC we have $\sphericalangle A \cong \sphericalangle B$, then ΔABC is isosceles.



Suggestion: Show again that $\Delta ABC \cong \Delta BAC$

ASA criterion for congruence

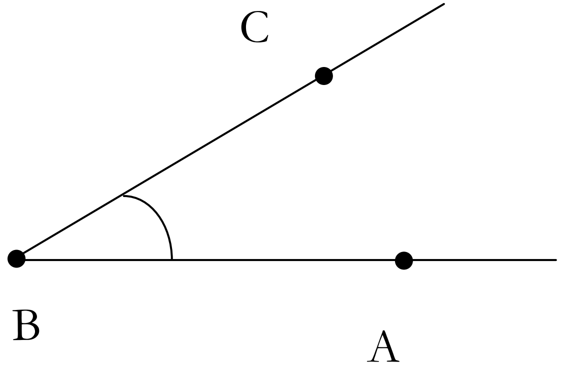
Proposition 3.17: Let ΔABC and ΔDEF be two triangles. If $\sphericalangle A \cong \sphericalangle D$, $\sphericalangle C \cong \sphericalangle F$ and $AC \cong DF$, then $\Delta ABC \cong \Delta DEF$.

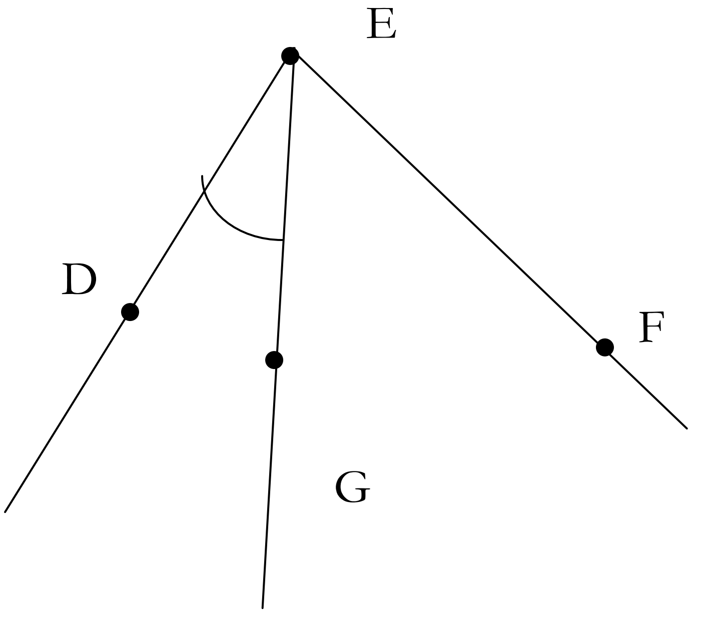
Sketch of proof:

- By axiom C1 there exists a unique point B' on $\overrightarrow{DE}$ st $DB' \cong AB$. We now have $\sphericalangle A \cong \sphericalangle D$, $AC \cong DF$ and $AB \cong DB'$ from which, using SAS, we conclude that $\Delta ABC \cong \Delta DB'F$. From the definition of congruent triangles we know that corresponding angles are congruent, hence $\sphericalangle DFB' \cong \sphericalangle C$. By axiom C4 we know that there is a unique ray $\overrightarrow{FX}$ on the side(B', $\overleftrightarrow{DF}$) such that $\sphericalangle C \cong \sphericalangle XFD$. Since both $\overrightarrow{FE}$ and $\overrightarrow{FB'}$ are such rays, we conclude that $\overrightarrow{FE}=\overrightarrow{FB'}$. Since both points E and B' lie on distinct, nonparallel lines $\overleftrightarrow{DF}$ and $\overleftrightarrow{DE}$, they must be the same point (Proposition 2.1), hence E=B'. Since $\Delta ABC \cong \Delta DB'F$, we conclude: $\Delta ABC \cong \Delta DEF$

Comparison

- ∢ ABC < ∢ DEF means there is a ray EG between ED and EF such that ∢ ABC ≅ ∢ DEG.





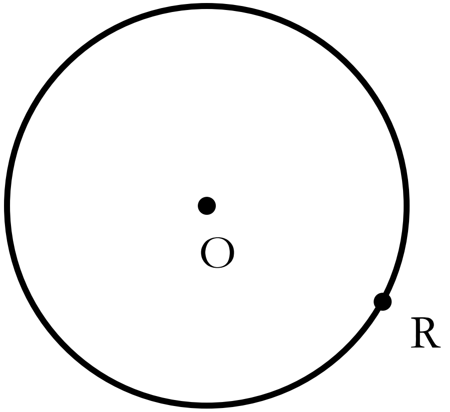
Proposition 3.21 (Ordering of angles)

- Proposition 3.21 (Ordering of angles)
  1. Exactly one of the following happens: $\sphericalangle P < \sphericalangle Q$, $\sphericalangle P \cong \sphericalangle Q$, or $\sphericalangle Q < \sphericalangle P$.
  2. If $\sphericalangle P < \sphericalangle Q$ and $\sphericalangle Q \cong \sphericalangle R$, then $\sphericalangle P < \sphericalangle R$.
  3. If $\sphericalangle P < \sphericalangle Q$ and $\sphericalangle P \cong \sphericalangle R$, then $\sphericalangle R < \sphericalangle Q$.
  4. If $\sphericalangle P < \sphericalangle Q$ and $\sphericalangle Q < \sphericalangle R$, then $\sphericalangle P < \sphericalangle R$.

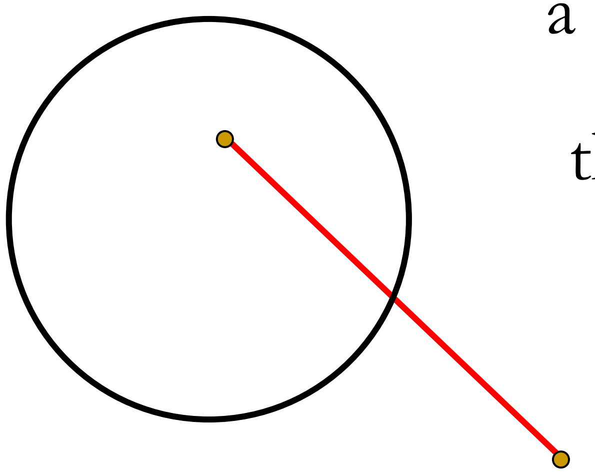
- Proposition 3.22 (SSS): Let ΔABC and ΔDEF be two triangles. If $AB \cong DE$, $BC \cong EF$ and $AC \cong DF$, then $\Delta ABC \cong \Delta DEF$.
- Proposition 3.23: All right angles are congruent.

New axioms and new terms

- Let O and R be two distinct points. The set of all points A such that OA ≅ OR is called a *circle*. Point O is called a *center*. Each segment OA is called *radius*. If B is a point such that OB<OR then B is *inside* the circle. If OB>OR then B is *outside* the circle.



Elementary continuity principle



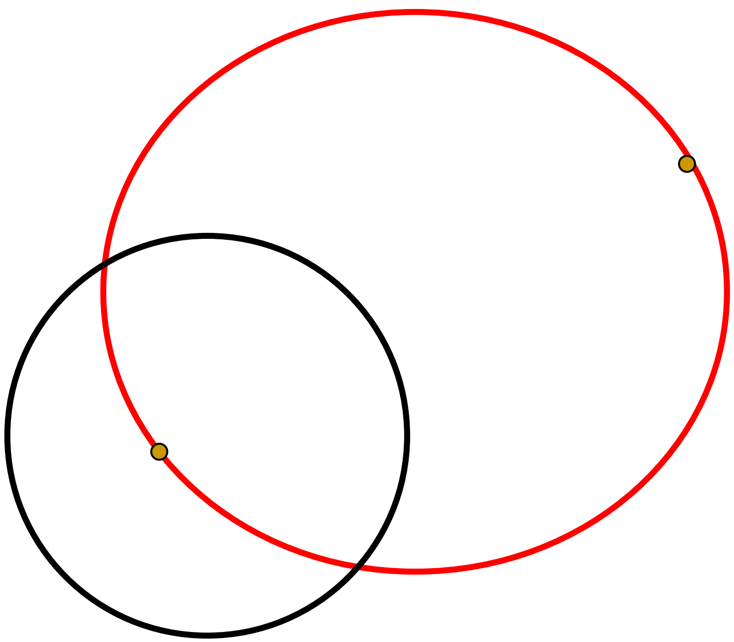
If one endpoint of a segment is inside a circle and the other outside

then the segment intersects the circle.

Circular continuity principle

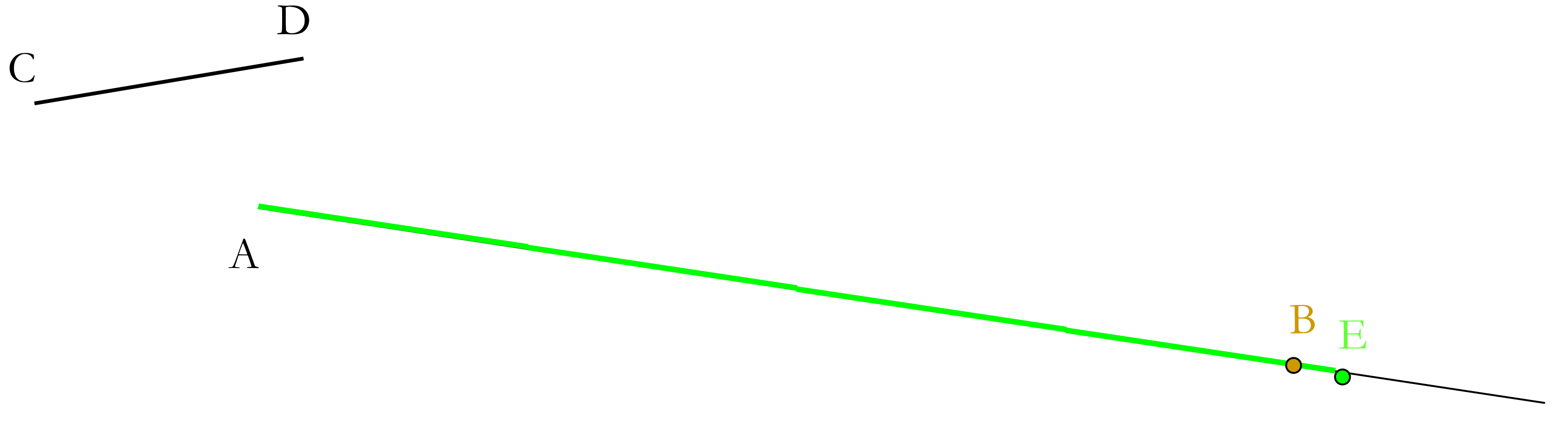
If a circle *c* has one point inside and one point outside another circle *c'*

then the two circles intersect in two points.



Measurements?

- Archemede's axiom: If CD is any segment, A any point and r any ray with vertex A, then for every point B $\neq$ A on r there is a number n such that when CD is laid off n times on r starting from A, a point E is reached such that $n \cdot CD \cong AE$ and either B=E or B is between A and E.



Given a segment OI called unit segment there is a unique way of assigning a length $\ell(AB)$ to each segment AB so that the following holds:

1. $\ell(AB)$ is a positive real number and $\ell(OI)=1$
2. $\ell(AB)= \ell(CD)$ iff $AB \cong CD$
3. A*B*C iff $\ell(AC)= \ell(AB)+ \ell(BC)$
4. $\ell(AB)< \ell(CD)$ iff $AB < CD$
5. For every positive real number x, there exists a segment AB such that $\ell(AB)= x$.

There is a unique way of assigning a degree measurement to each angle so that the following holds:

1. $m(\sphericalangle A)$ is a real number such that $0< m(\sphericalangle A)<180^o$
2. $m(\sphericalangle A)=90^o$ iff $\sphericalangle A$ is a right angle.
3. $m(\sphericalangle A)= m(\sphericalangle B)$ iff $\sphericalangle A \cong \sphericalangle B$.
4. If C is in the interior of $\sphericalangle DAB$ then $m(\sphericalangle DAB)= m(\sphericalangle DAC)+m(\sphericalangle CAB)$
5. For every real number x between 0 and 180 there is an angle $\sphericalangle A$ such that $m(\sphericalangle A) =x^o$
6. If $\sphericalangle B$ is supplementary to $\sphericalangle A$, then $m(\sphericalangle A)+m(\sphericalangle B)=180^o$
7. $m(\sphericalangle A)>m(\sphericalangle B)$ iff $\sphericalangle A > \sphericalangle B$