Class #22

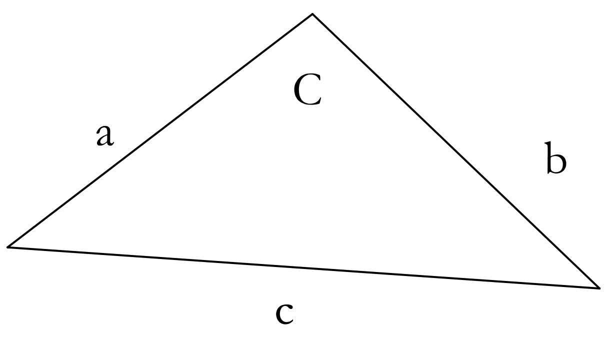
Congruence …

Model

- In the Cartesian plane we can define length of segments:
  - A=(a_1, a_2) and B=(b_1, b_2), then

- Two segments are congruent iff they have equal length.
- Two angles are congruent iff they have equal measures.

- $c^2 = a^2 + b^2 – 2ab\cdot cos\ C$, where



SAS is independent of other axioms

- Use Cartesian plane, except the length of the segments whose endpoints are on the x-axis will be twice as long as they used to be.
- Does this satisfy ***C1-5***? What about ***SAS***?

Proposition 3.11 (Segment subtraction)

- If A*B*C and D*E*F, AB ≅ DE and AC ≅ DF, then BC ≅ EF.

- Proof on next homework.

Proposition 3.12. If $AC \cong DF$, then for any point B between A and C, there is a unique point E between D and F such that $AB \cong DE$.

- Sketch of proof: By ***C1*** there is a unique point E on $\overrightarrow{DF}$ such that $AB \cong DE$. Either:
  - D*E*F – done
  - E=F – we have $AB \cong DE$ and $AC \cong DE$, hence by ***C3*** we have $AB \cong AC$. Since B≠C, we have a contradiction to uniqueness in ***C1***.
  - D*F*E – then on the ray opposite to $\overrightarrow{CA}$ there is a unique point G such that $CG \cong EF$. By ***C3***, $AG \cong DE$. Further, since $AB \cong DE$ we have that $AB \cong AG$. We know that B≠C (since B*C*G by Prop. 3.3), again contradicting the uniqueness in ***C1***.

Comparing segments?

- Is there any way of telling when one segment is "smaller" then another without talking about length? Use only undefined terms we have.

- *Definition:* $AB<CD$ if there exists a point E between C and D such that $AB \cong CE$.

- When is $AB \not< CD$?

To contemplate for Friday:

Proposition 3.13:

1. Exactly one of the following holds: $AB < CD$, $AB \cong CD$, or $AB > CD$.
2. If $AB<CD$ and $CD \cong EF$, then $AB<EF$.
3. If $AB<CD$ and $AB \cong EF$, then $EF<CD$.
4. If $AB<CD$ and $CD<EF$, then $AB<EF$.