



### **GEOMETRY**



### Emina

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### Back to business

#### Class website

#### http://www.math.lsa.umich.edu/~eminaa/teaching/431



### Euclidean and non-Euclidean Geometries: History and Development

by Greenberg (3<sup>rd</sup> edition)

# Help

#### • Office hours

- Monday 10:30 11:30
- Wednesday 2:30 3:30
- Friday 10:30 11:30
- By appointment email me!



- 30% Homeworks
- 20% Exam 1
- 20% Exam 2
- 30% Final

### Homeworks

- No late homeworks
- No extensions
- Write in complete sentences
- Listed on the website
- READ EACH SECTION IN ADVANCE
  - This will greatly increase your understanding and success.
     Consequences for failure to do so include ...
- First one due on 9/16.

### Exams

Midterm 1
October 6 – in class
Midterm 2
November 10 – in class
Final
???

# Possible prejudices

- IS NOT a class about how to TEACH geometry!!!
- IS a class about learning:
  Geometry: Euclidean, hyperbolic and spherical
  How to do mathematics rigorously
- May NOT fulfill your minor requirements

## However...

- Most of the material we cover you will be teaching at some point, and a lot more than that
- We will not cover the whole geometry curriculum, but you will be able to work your way through it
- We will try to consult geometry textbooks and see how we cope with the problems within.

# Ultimate goal

If you do not already, by the end of semester you should love and appreciate the beauty and powers of reason and geometry.

 The only great teachers are the ones with great love and knowledge of their subjects.

## Euclid's view:

- Student:
  - □ What shall I get by learning these things?
- Euclid to his slave:
  - Give him a coin, since he must make gain out of what he learns!

# OUTLINE

- Axiomatic method, logic, sets, proofs.Test our understanding on incidence geometry
- Hilbert's axioms and lots of theorems
- Less focus on axioms and more on models and analytic approach. Time permitting discussions of various applications.

## Quick ride through history

geo – earth

#### Geometrein

metrein – to measure

# Ancient history...

- Ancient geometry
  - Empirical results obtained through experimentation, observation, analogies, guessing
  - Often correct, but sometimes not



### Greek geometry (~ 600 BCE)

Insisted on deductive reasoning

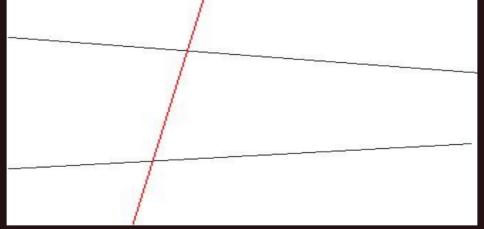
- Logical geometry
- □ Euclid's *Elements* (~ 300 BCE)

### Euclid's Postulates

- To draw a straight line from any point to any point.
- To extend a finite straight line continuously in a straight line.
- To describe a circle with any center and radius
- That all right angles are equal to each other
- That if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the straight line, if produced indefinitely, meet on that side on which are the angles less than two right angles.

# The Fifth Postulate

That if a straight line falling on two straight lines makes the interior angles on the same side less than two right angles, the straight line, if produced indefinitely, meet on that side on which are the angles less than two right angles.



For every line *l* and every point *P* not on *l*, there exists a unique line that contains *P* and is parallel to *l*.

## Next 2000 years

- Euclid's work mostly accepted as was
- Controversy: 5<sup>th</sup> postulate: Is it or is it not?
- Bolyai, Lobachevski and Gauss in 1800 settled the matter!!!

# What is geometry?

### Greeks thought:

□ True and absolute description of physical world

#### • We think:

 Statements are either true or false GIVEN CERTAIN HYPOTHESES

 We give hypotheses and discover their consequences: theorems.

## Poincaré

If geometry were an experimental science, it would not be an exact science. It would be subject to continual revision.. The geometrical axioms are therefore neither synthetic a priori intuitions nor experimental facts. They are conventions. Our choice among all possible conventions is guided by experimental facts; but it remains free, and is only limited by necessity of avoiding every contradiction... What then are we to think of the question: Is Euclidean geometry true? It has no meaning. We might as well ask if the metric system is true and if the old weights and measurements are false; if Cartesian coordinates are true and polar coordinates false. One geometry cannot be more true than another: it can only be more convenient.

Procedure by which we demonstrate or **prove** that statements are indeed correct (given hypotheses).

- What is a statement?
- What is a consequence?
- What do we mean by true?

# Thought experiment!!!

Figure out the meaning of each statement and then determine whether it is *true* or *false*.

Try to justify your answer. If you can't give an answer, say why you can not.

Don't get bogged down by a single problem.