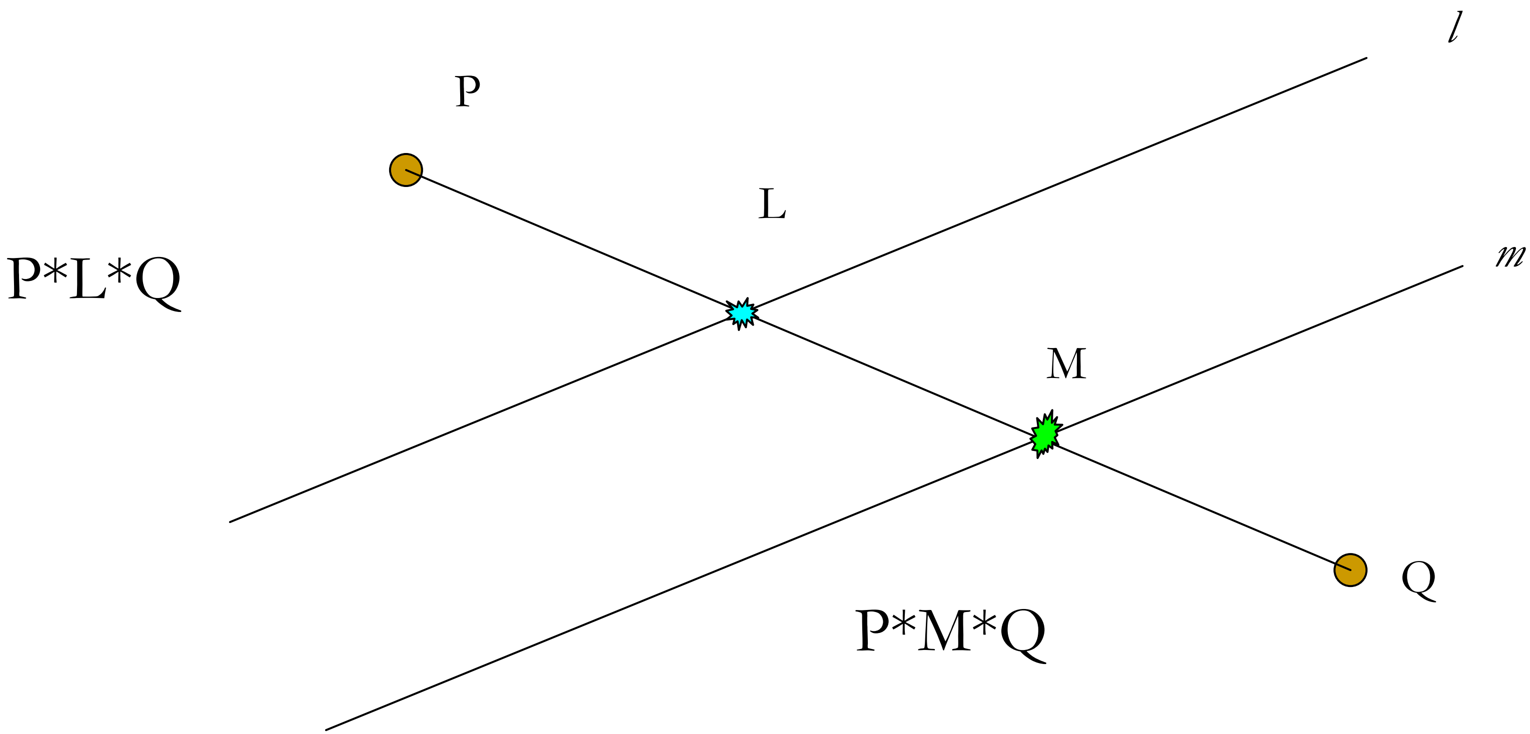
Class #17

LSP

Question – will become a hw6#1:

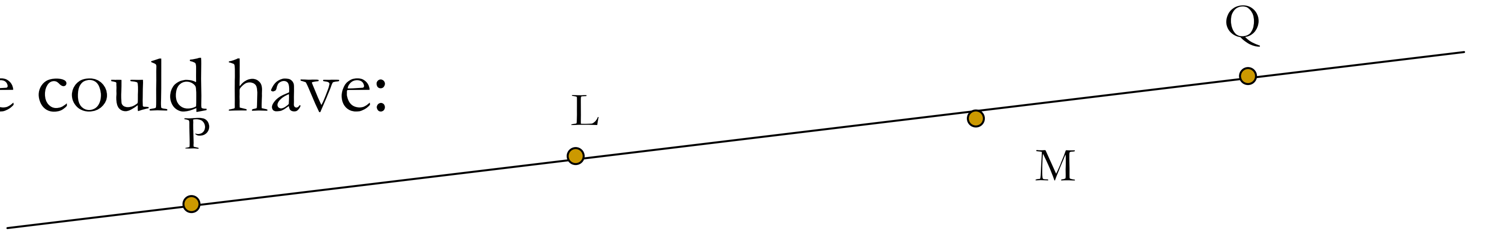
- If side(P, l) ∩ side(Q, m)=∅, what can you say about P, Q, l and m?



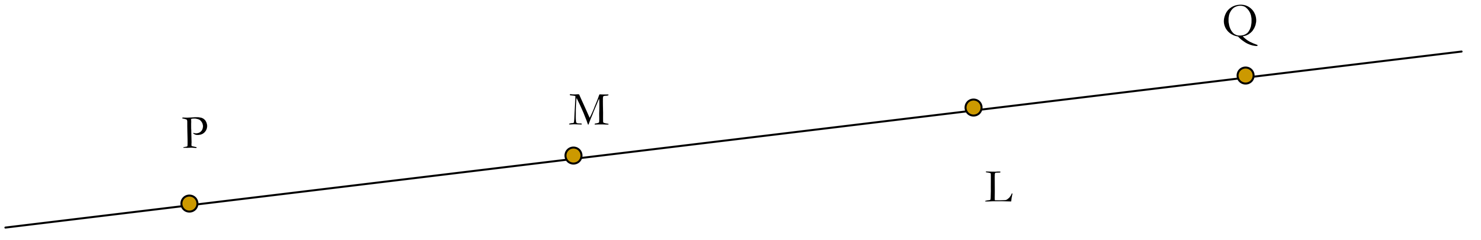
In this case we can conclude P*L*M & L*M*Q

If P*L*Q and P*M*Q the following picture tells us that in general we can not claim that P*L*M and L*M*Q.

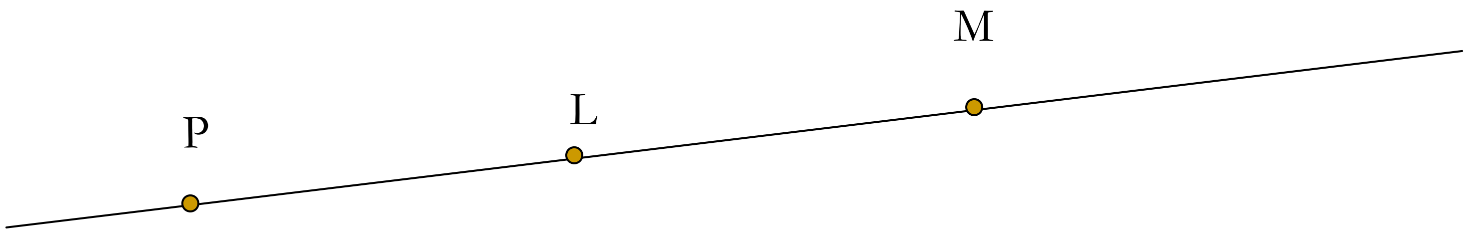
We could have:



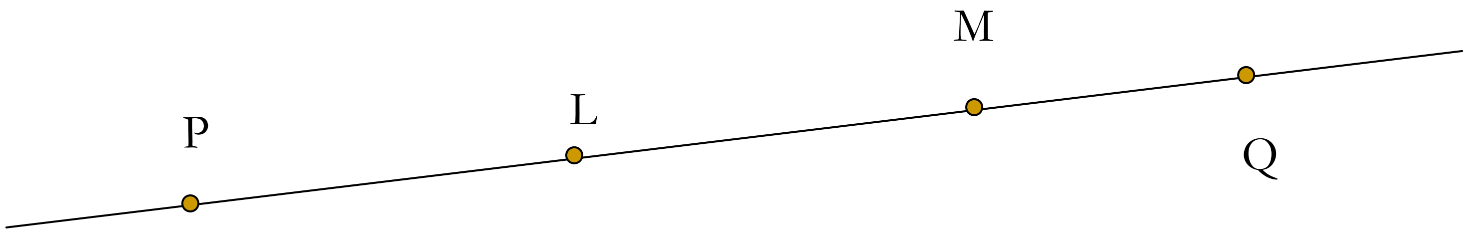
or



If P*L*M



If P*L*M and P*M*Q



then P*L*Q and L*M*Q

Proposition 3.3: If A*B*C and A*C*D then B*C*D and A*B*D.

- By Hw4.1 we know that A, B, C and D are four distinct, collinear points. Let *l* be the line they all lie on. By Proposition 2.3. there is a point E not lying on *l*. Since E is not on *l* and C is, these two points must be distinct. Let $\overleftrightarrow{EC}$ be the line through E and C (whose existence and uniqueness is guaranteed by ***I-1***). Note that $\overleftrightarrow{EC}$ and *l* are two distinct lines. Since AD meets $\overleftrightarrow{EC}$ in a point C and A*C*D we conclude that A and D are on opposite sides of $\overleftrightarrow{EC}$. Further, A and B are on the same side of $\overleftrightarrow{EC}$. If not, then AB intersects $\overleftrightarrow{EC}$ in a point, say C'. Note that A*C'*B. C' lies both on *l* and on $\overleftrightarrow{EC}$. Since C also lies on both those lines, by Proposition 2.1 we have C=C'. We hence have A*C*B and, by hypothesis, A*B*C, which contradicts ***B-3***. Therefore, A and B are on the same side of $\overleftrightarrow{EC}$. This together with A and D on opposite sides of $\overleftrightarrow{EC}$ gives that B and D are on opposite sides of $\overleftrightarrow{EC}$ (by Corollary to ***B-4***). Therefore BD intersects $\overleftrightarrow{EC}$, and using the same argument as above we conclude that it intersects it in C, hence B*C*D.

LSP: If C*A*B and *l* is the line through A, B and C, then any point P lying on *l* lies either on $\overrightarrow{AB}$ or on $\overrightarrow{AC}$.

- Let P be a point lying on *l*. If $P \in \overrightarrow{AB}$, the claim follows. If it is not (in which case P*A*B), then we would like to show that $P \in \overrightarrow{AC}$. Suppose the contrary, that is P*A*C. Let us consider where P could be in relation to C and B. By B3 we have that one of the following holds: P*C*B or C*P*B or C*B*P.
  - If P*C*B, then by Proposition 3.3 since P*A*C, we get A*C*B, which contradicts our hypothesis.
  - If C*P*B, then by Proposition 3.3 since P*A*C, we get A*P*B, and P lies on the $\overrightarrow{AB}$, which contradicts our assumption.
  - If C*B*P, we use P*A*B to conclude that C*B*A, which contradicts our hypothesis.

Our assumption P*A*C led to contradiction, therefore it must be that A*P*C or A*C*P, and P is on $\overrightarrow{AC}$.