Axioms

Incidence axioms

I-1: For any two distinct points there exists a unique line that passes through both of them.

I-2: For any line there exist at least two distinct points incident with it.

I-3: There exist three distinct points with the property that no line is incident with all three of them.

Betweenness axioms

B-1 If $A \star B \star C$, then A, B and C are three distinct points all lying on the same line, and $C \star B \star A$.

B-2 Given any two distinct points B and D, there exist points A, C, and E lying on $\overleftrightarrow{B}D$ such that $A \star B \star D$ and $B \star C \star D$, and $B \star D \star E$.

B-3 If A, B, and C are three distinct points lying on the same line, then one and only one of the points is between the other to.

B-4 For every line ℓ and for any three points A, B, and C not lying on ℓ:

(i) If A and B are on the same side of ℓ and B and C are on the same side of ℓ, then A and C are on the same side of ℓ.

(ii) If A and B are on opposite sides of ℓ and B and C are on opposite sides of ℓ, then A and C are on the same side of ℓ.

Congruence axioms

C-1 If A and B are distinct points and if A' is any point, then for each ray r eminating from A' there is a unique point B' on r such that $B' \neq A'$ and $AB \cong A'B'$.

C-2 If $AB \cong CD$ and $AB \cong EF$, then $CD \cong EF$. Moreover, every segment is coungruent to itself.

C-3 If $A \star B \star C$, $A' \star B' \star C'$, $AB \cong A'B'$ and $BC \cong B'C'$, then $AC \cong A'C'$.

C-4 Given any angle $\sphericalangle BAC$, and given any ray $\overrightarrow{A}'B'$ emanating from a point A', there is a unique ray $\overrightarrow{A}'C'$ on a given side of the line $\overleftrightarrow{A}'B'$ such that $\sphericalangle B'A'C' \cong \sphericalangle BAC$.

C-5 If $\sphericalangle A \cong \sphericalangle B$ and $\sphericalangle A \cong \sphericalangle C$, then $\sphericalangle B \cong \sphericalangle C$. Moreover, every angle is congruent to itself.

C-6 (SAS) If two sides and the included angle of one triangle are congruent respectively two two sides and the included angle of another triangle, then the two triangles are congruent.

Parallelism axioms

Euclidean PP For every line l and every point P not on l there exists a unique line m passing through P parallel to l.

Elliptic PP For every line l and every point P not on l there is no line passing through P parallel to l.

Hyperbolic PP For every line l and every point P not on l there exist at least two distinct lines passing through P parallel to l.
