Notations and definitions

There are certain terms that I will assume you know. Here is a refresher sheet.

1 Sets

A set is a collection of things of any kind. If B is a set we call the "things" in B the elements or members of B.

$b \in B$ reads 'b is an element of B'

$b \notin B$ reads 'b is not an element of B'

Suppose A and B are two sets.

We say that A is a *subset* of B, and we write $A \subset B$, if every element of A is also an element of B.

$A \cup B$ is called the *union* of A and B and it consists of all elements that are either in A or in B:

$$A \cup B = \{x \mid x \in A \text{ or } x \in B\}$$

$A \cap B$ is called the *intersection* of A and B and it consists of all elements that belong to both A and B:

$$A \cap B = \{x \mid x \in A \text{ and } x \in B\}$$

$A \backslash B$ is called the *difference* of A and B and it consists of all elements that belong to A, but not to B:

$$A \backslash B = \{x \mid x \in A \text{ and } x \notin B\}$$

We will almost always write sets in the set notation. For example, the set of all even integers, $2\mathbb{Z}$, can be written as follows:

$$2\mathbb{Z} = \{x \in \mathbb{Z} \mid x = 2y, \text{ for some } y \in \mathbb{Z}\}$$

Sets that are important to us:

$\mathbb{N} = \{1, 2, 3, 4, 5, 6, 7, \ldots\}$	natural numbers
$\mathbb{Z} = \{x \mid x = 0 \text{ or } x \in \mathbb{N} \text{ or } -x \in \mathbb{N}\}$	integer numbers
$\mathbb{Q} = \{x \mid \exists p \in \mathbb{Z}, q \in \mathbb{Z} \backslash \{0\}\ x = \frac{p}{q}\}$	rational numbers

However, there are numbers that are not rational, i.e. they can not be represented as a fraction. These numbers are called *irrational*. The union of all rational and irrational numbers is called the set of *real* numbers, $\mathbb{R}$. Certain subset of $\mathbb{R}$ appear often and warrant getting special names:

$$(a, b) = \{x \in \mathbb{R} \mid a < x < b\} \qquad \text{open interval}$$

$$[a, b] = \{x \in \mathbb{R} \mid a \leq x \leq b\} \qquad \text{closed interval}$$

$$[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\} \qquad \text{semi-open interval}$$

2 Functions

Let X and Y be two sets. A *function* f from X to Y, $f : X \to Y$, is an assignement that to each element $x \in X$ assigns a unique element of Y, $f(x)$, called the value of f at x. X is called the *domain* of f, and Y is its *codomain.* The *range (image)* of f is the set of all possible values of f:

$$Im(f) = f(X) = \{y \in Y \mid \exists x \in X \ y = f(x)\}$$

In general, $Im(f) \subset Y$, but it those two sets are equal we say that f is *onto (surjective).* Equivalently, if the equation $f(x) = y$ has at least one solution for each $y \in Y$, f is onto.

f is *1-1 (injective)* if $f(x) = f(z) \implies x = z$. Equivalently, f is 1-1 if the equation $f(x) = y$ has at most one solution for each $y \in Y$.

If f is both 1-1 and onto, that is if the equation $f(x) = y$ has EXACTLY one solution for each $y \in Y$, we say that f is invertible, in which case there is a function $f^{-1} : Y \to X$ such that:

$$f^{-1}(f(x)) = x \ \ \forall x \in X$$

$$f(f^{-1}(y)) = y \ \ \forall y \in Y$$

If $g : Y \to Z$ is another function we form the *composition* of f and g, $g \circ f$, which is defined as follows:

$$(g \circ f)(x) = g(f(x)) \ \ \forall x \in X$$