Number theory cont'd

- If a whole number is divisible by both 2 and 3 does it mean it’s divisible by 6?
- If a whole number is divisible by 4 and 6 is it divisible by 24?
- If a whole number W is divisible by a and b, and a and b have no common factors then W is divisible by $a \cdot b$.

Back to our example

- Is 401 prime or not? If it is not, what is its prime factorization?
- What about 5439?
- What about 15249?

  - Not divisible by 2, 5
  - What about 3? 7? 11? 13? 17?

Divisibility by 3

- A whole number is divisible by 3 if the sum of its digits is divisible by 3.

- Proof:

  - $3 \mid 401$
  - $3 \nmid 5439$
  - $3 \mid 15249$

401!@!*$#@^%#$!(^%

- What about this 401? How far should we go?

- I think once we’re past 20 all hope is gone and we should declare it prime! Am I right?

Hmmmm…. How many primes are there?

Grid rectangle problem

- How many grid rectangles are there for number 84?
- What about any number m?

Problem #1

- Pencils come in packages of 18; erasers that fit on top of these pencils come in packages of 24. What is the smallest number of pencils and erasers that you can buy so each pencil can be matched with an eraser? How many packages of each will you need?

Least common multiple

- The LCM of two (or more) nonzero numbers is the least whole number that is a multiple of each of the numbers.
- LCM(18,24)=72

Problem #2

- Ko has a bag with 45 red candies and another with 75 green candies. She wants to make goody bags so that each goody bag contains the same number of red candies and each goody bag contains the same number of green candies and so that she uses up all of the candies. What is the largest number of goody bags she can make this way? How many of each color will be in the bag?

Greatest common factor

- The GCF of two (or more) nonzero numbers is the largest whole number that is a factor of both (all) numbers.

- GCF(45,75)=15

Finding LCM and GCF

- Let’s look at 36 and 28.

- Multiples of 36 are {36, 72, 108, 144, 180, 216, 252, …, 1008,...}
- Multiples of 28 are {28, 56, 84, 112, 140, 168, 196, 224, 252, …, 1008, ….}

- Common multiples are {252, 504, 756, 1008, ….}

- The SMALLEST one is 252 $\Rightarrow$ LCM(28, 36)=252.

LCM

- $36=2^2 \cdot 3^2$
- $28=2^2 \cdot 7$
- If we pick up all the factors, without repeating ones that appear in both numbers, we'll get a multiple of each number:

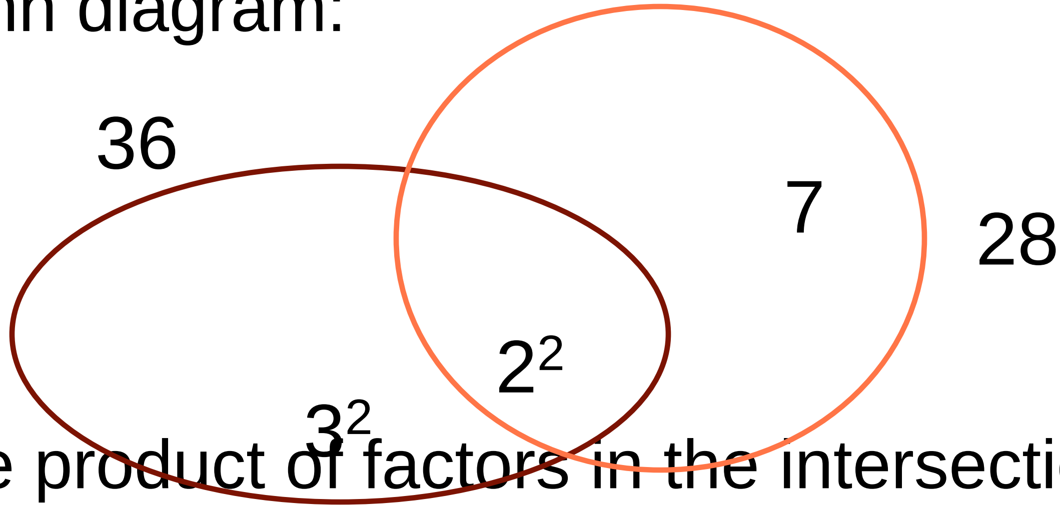
$$7 \cdot (2^2 \cdot 3^2)=7 \cdot 36=252$$

$$(7 \cdot 2^2) \cdot 3^2 =28 \cdot 9=252$$

Notice that we found GCF(28,36) to be 2^2

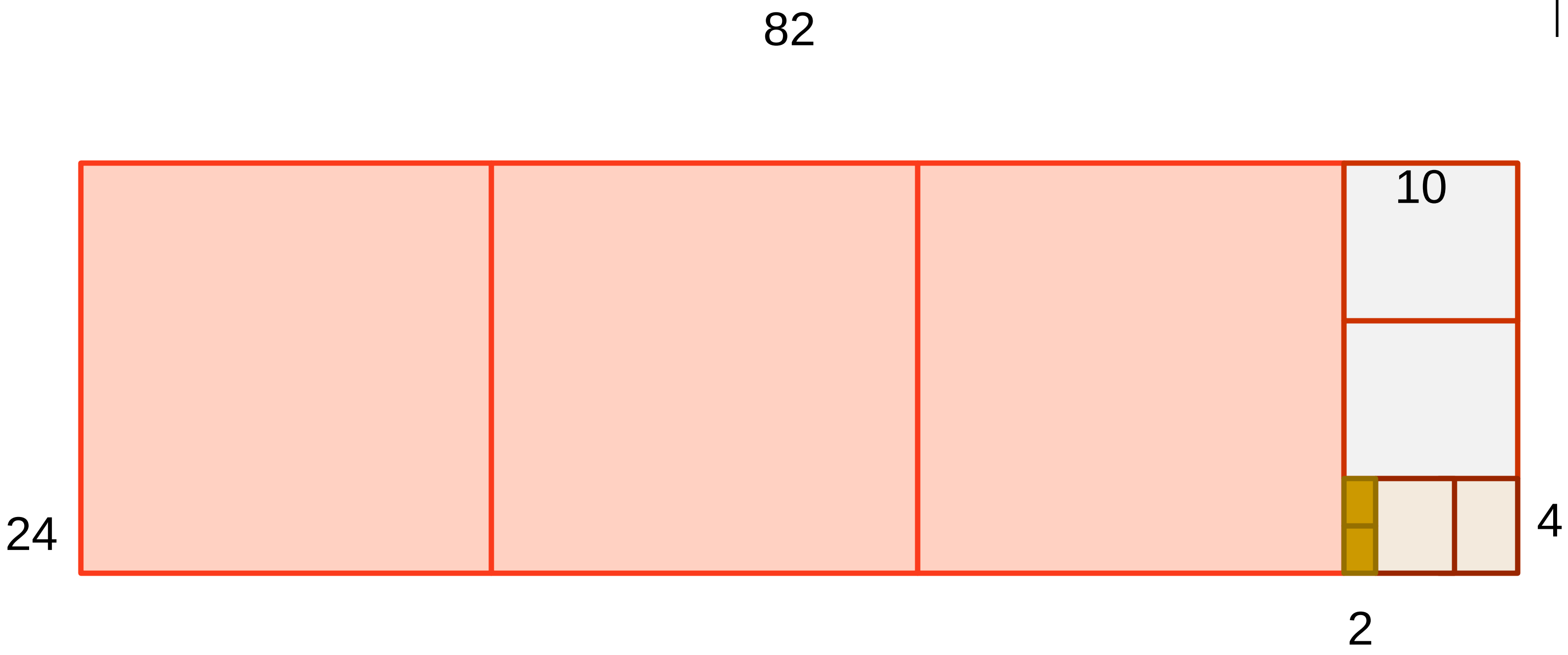
One way to find both GCF and LCM is to use Venn Diagrams

- Write prime factorization of each number. Draw a Venn diagram:



- The product of factors in the intersection is their GCF: 2^2
- The product of factors in the union is their LCM: $7 \cdot (2^2 \cdot 3^2)$

GFC(24, 82)



Another way

	28	36	

Euclid's Algorithm

- This picture corresponded to

$82=3\cdot 24+10$

$24=2\cdot 10+4$

$10=2\cdot 4+2$

$4=2\cdot \boldsymbol{2}$

Proofs in elementary mathematics

- Proofs establish that something is true
- Proofs can explain why something is true
- Proof requires establishing something is true for all cases
- Giving examples can illustrate or explore a claim but is not sufficient for a mathematical proof
- Even a lot of evidence for a claim is not enough to know that it is true
- A counterexample is sufficient to show that a claim is false

Proving is part of teachers' work

- Teachers must recognize when a statement or claim needs a proof
- Teachers needs to be attentive to what it means to explain something or show something to be true mathematically
  - Teachers must explain mathematical ideas with integrity
  - Students sometimes make claims that are not clearly true or false and teachers must sort these out
  - Teachers must consider possible counterexamples to claims that students make
- Teachers must help students learn to prove mathematical statements

Why are definitions necessary in mathematics?

- Mathematics is a domain in which meanings are precise and not connotative
- Most mathematical terms are also used in other ways more generally m*(“even”)*
- Being clear about mathematical meanings makes effective communication possible
- Mathematical reasoning depends on definitions

How do definitions and defining show up in teaching?

- In class, listening carefully to students' talk, noticing and helping improve ambiguous talk, including mediating everyday uses of language
- Analyzing and managing disagreements for their possible roots in different or imprecise uses of terms
- Recognizing, comparing, and reconciling different definitions
- Considering how to define terms so that they are usable with an eye toward the future
- Defining terms carefully for or with students
- Scrutinizing definitions in curriculum materials and modifying as needed
- Making sure that students can interpret test items