Number Theory

Primes, factorization, divisibility rules

“divisible by”

- Let a and b be any whole numbers and $a \neq 0$. We say that a divides b, $a|b$, if there is a whole number x such that $ax=b$.
- The following mean the same thing:
  - a divides b,
  - b is divisible by a,
  - a is a factor of b,
  - b is a multiple of a

Our homework problem #4

- 1001|abcabc
- 7, 11, 13|1001,

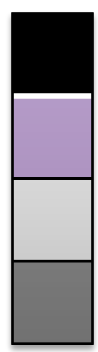
So it must be that 7, 11, 13|abcabc

Claim: If d is not 0 and d|e then d|ef.

Grid rectangle problem

- For numbers of tiles 1 through 24 build all grid rectangles that you can.
- How do you know you have all of them?
- What patterns do you notice?

1x4



4x1

Claims

Primes and composites

- A counting number with exactly 2 different factors is called a *prime number.*
- A counting number with more than two factors is called a *composite number.*

  - Examples of prime numbers:

  - Examples of composite numbers:

What do you notice?

1	2	3	4 =2·2	5	6 =2·3	7	8 =2·2·2	9 =3·3	10 =2·5
11	12 =2·2·3	13	14 =2·7	15 =3·5	16 =2·2·2 ·2	17	18 =2·3·3	19	20 =2·2·5
21 =3·7	22 =2·11	23	24 =2·2·2· 3	25 =5·5	26 =2·13	27 =3·3·3	28 =2·2·7	29	30 =2·3·5

Question:

- Can every number be written as product of primes?

- Solve simpler problem: Can 72 be written as product of primes?

Unique prime factorization

- Each composite number can be written as a product of primes in exactly one way (up to the order of factors).

- $72 = 2^3 \cdot 3^2$
- $81=3^4$

Food for thought

- Is 401 prime or not? If it is not, what is its prime factorization?
- What about 5439?
- What about 15249?

  - How would you approach this question?
  - How do we deal with large examples?
    - I know that none of these are divisible by 2, 5 or 10. How?

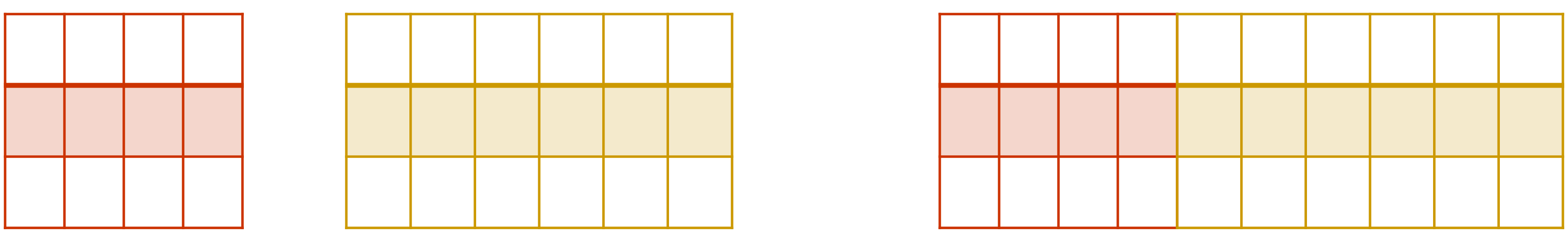
Divisibility by 2

- A whole number is divisible by 2 if its ones digit is 0, 2, 4, 6, or 8.

- Proof:

- Claim: If a number divides two other numbers then it divides their sum as well.

  - a|m and a|n, so a|(m+n)



Divisibility by 5

- A whole number is divisible by 5 if its ones digit is 0 or 5.

- Proof:

Divisibility by 10

- A whole number is divisible by 10 if its ones digit is 0.
- A whole number is divisible by 10 if it is divisible by both 2 and 5.

- If a whole number is divisible by both 2 and 3 does it mean it’s divisible by 6?

- If a whole number is divisible by 4 and 6 is it divisible by 24?

Food for thought

- Is 401 prime or not? If it is not, what is its prime factorization?
- What about 5439?
- What about 15249?

  - Not divisible by 2, 5
  - What about 3? 7? 11? 13? 17?

Divisibility by 3

- A whole number is divisible by 3 if the sum of its digits is divisible by 3.

- Proof:

  - $3 \nmid 401$
  - $3 \mid 5439$
  - $3 \mid 15249$

401!@!*$#@^%#$!(^%

- What about this 401? How far should we go?
- I think once we’re past 20 all hope is gone and we should declare it prime! Am I right?

Grid rectangle problem

- How many grid rectangles are there for any number m?