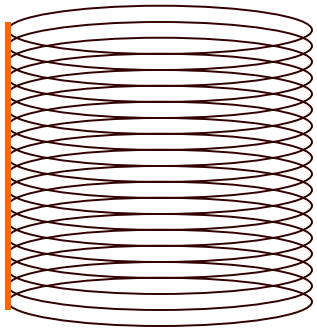
Products

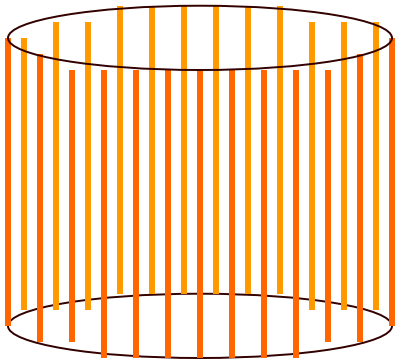
- Stack some circles!



Cylinder
-- interval of circles

Or maybe

- Stack some intervals!



Cylinder
-- circle of intervals

Cylinder

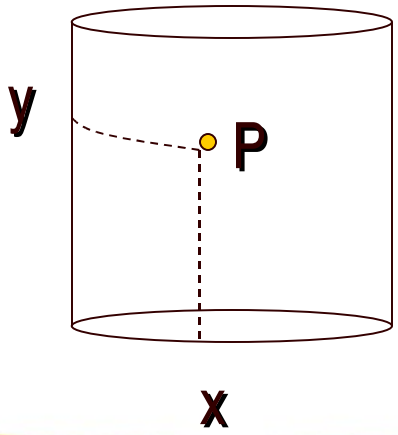
- Is a product of a circle and an interval

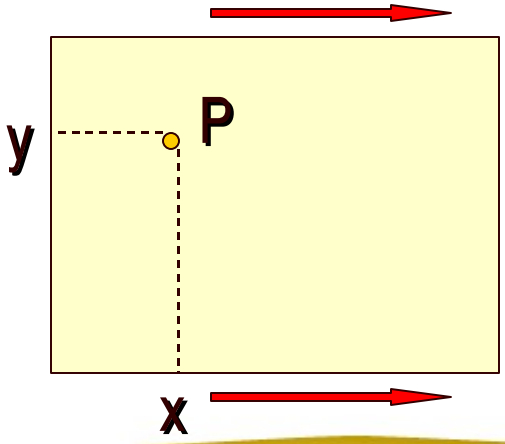
$$S^1 \times I$$

(circle cross interval)

In coordinates:

- Any point P on the cylinder can be given as (x,y), where x is a point on the circle and y is a point on the interval





Problem

- Can you think of a closed surface that is a product?
  - Torus – draw pictures
  - It is $\mathbf{S}^1 \times \mathbf{S}^1$
- Is that the only CLOSED surface which is a product?
  - Why?

Exercises

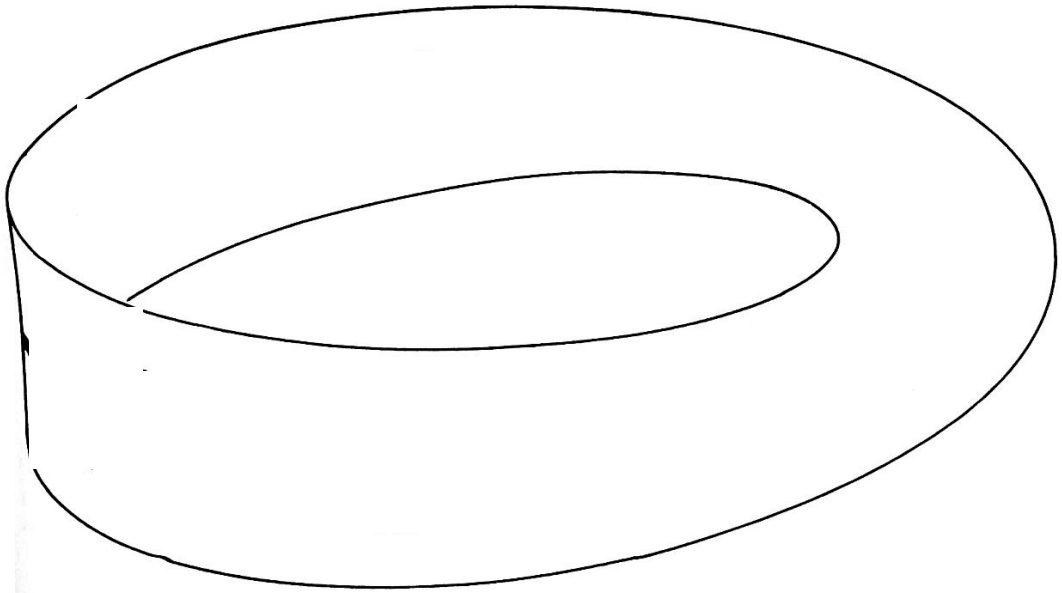
Draw pictures whenever possible.

- What is **I** x **I**?
- What is $\mathbf{E^1}$ **x I**?
- What is $\mathbf{E^1}$ **x** $\mathbf{E^1}$ **?**
- What is $\mathbf{E^1}$ **x** $\mathbf{S^1}$ **?**
- What is $\mathbf{D^2}$ **x** $\mathbf{S^1}$ **?**

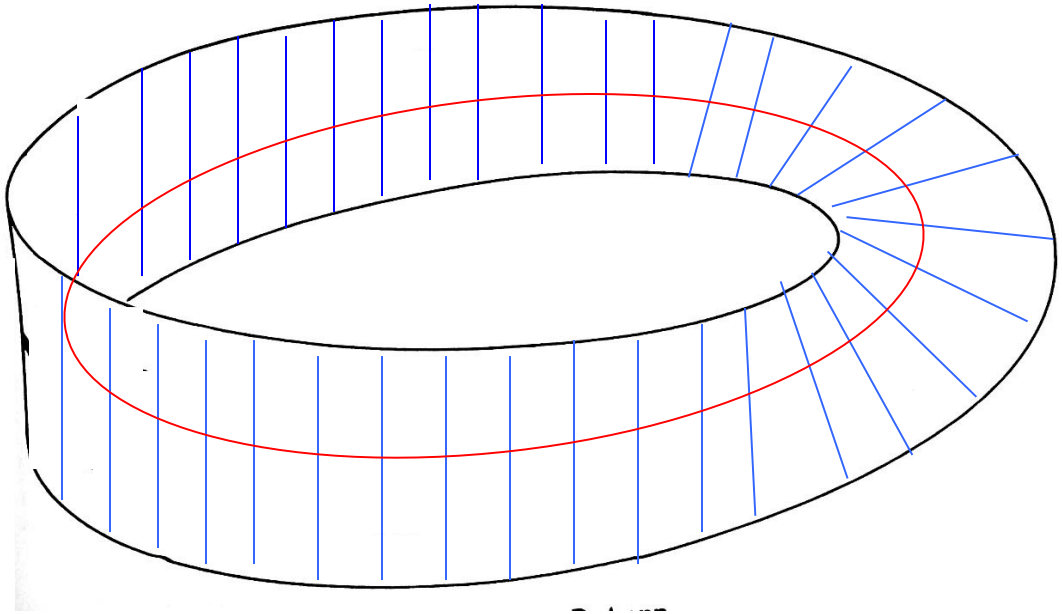
- **square**
- **infinite flat strip**
- $\mathbf{E^1}$
- **Infinite cylinder**
- **filled torus**

Exercise

- Is the Möbius strip a product?
  - No.

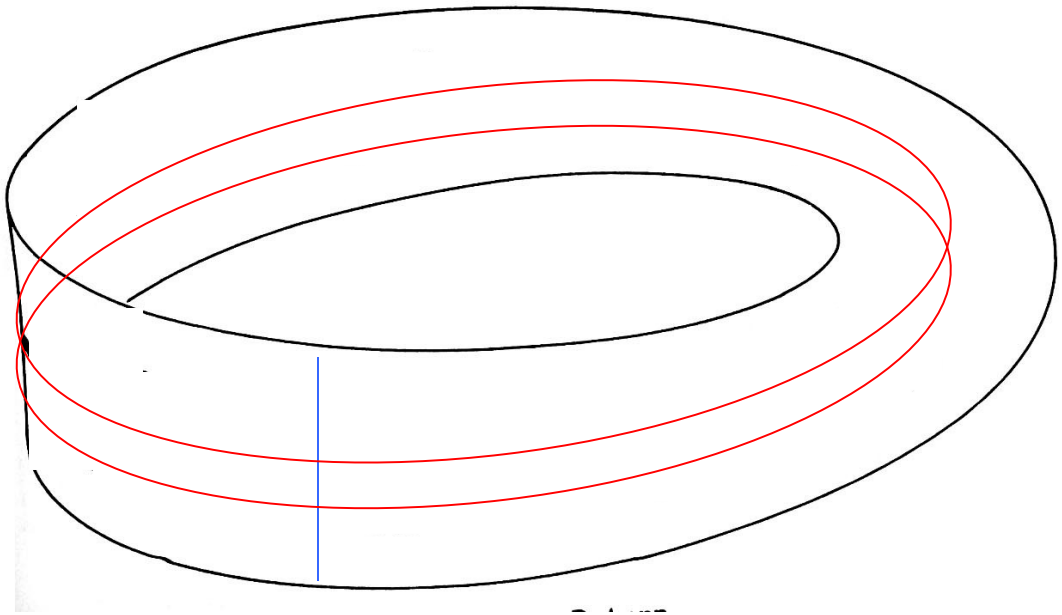


- It is a circle of intervals



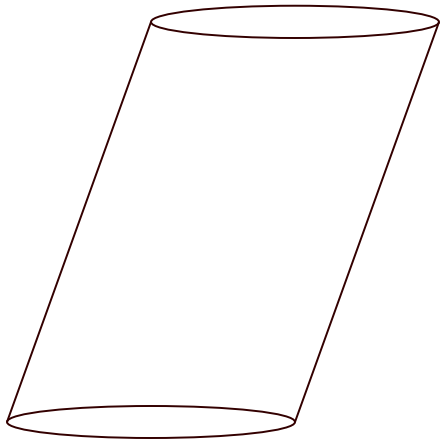
Exercise

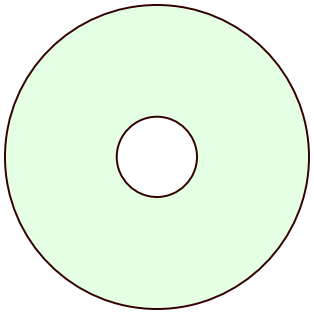
- It is not an interval of circles.

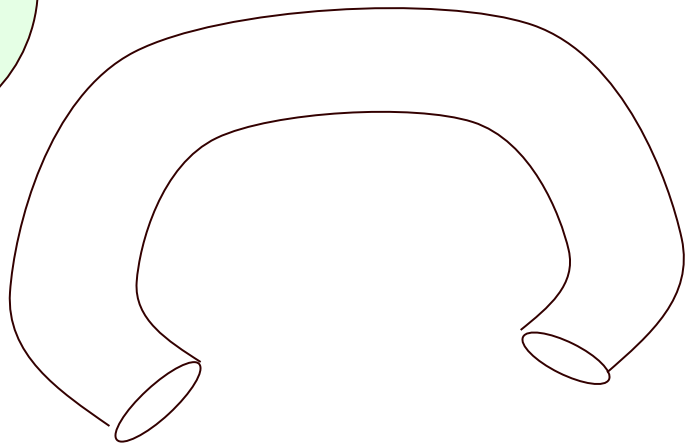


Different products

Are these all products?

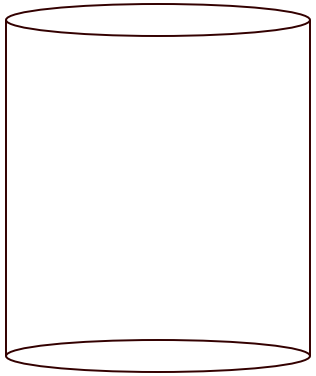






- They are all topological products
- But not geometric products

Geometric products

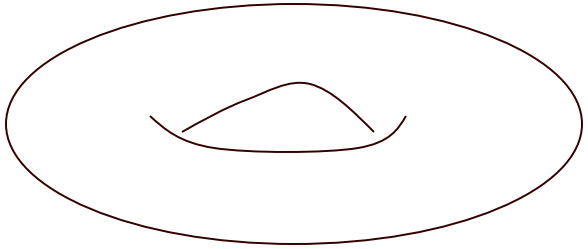


Is a geometric product because:

- all intervals have the same size
- all circles have the same size
- all intervals are perpendicular to all circles

Torus?

- Is this torus a geometric product?



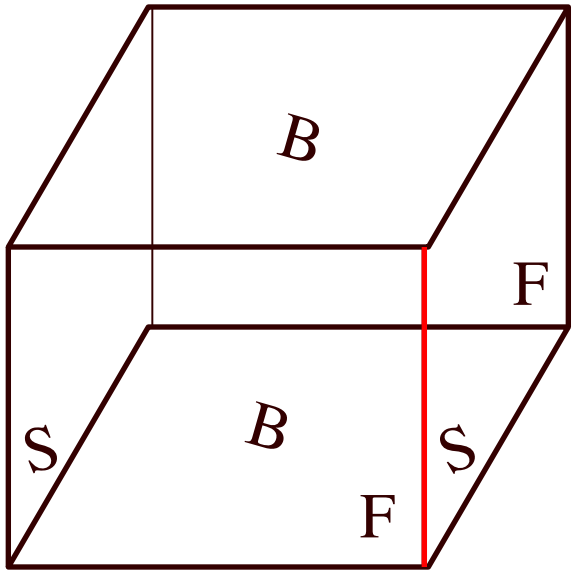
No

Question

- Can you make a geometric product of two circles in our 3-space?
  - **no – we can make the geometric cylinder, but then we need to connect top and bottom circle**
- What is a geometric $\mathbf{S}^1 \times \mathbf{S}^1$?
  - **flat torus – all the circles in one direction have the same size, as do all the circles in the other direction, and those directions are perpendicular to each other**

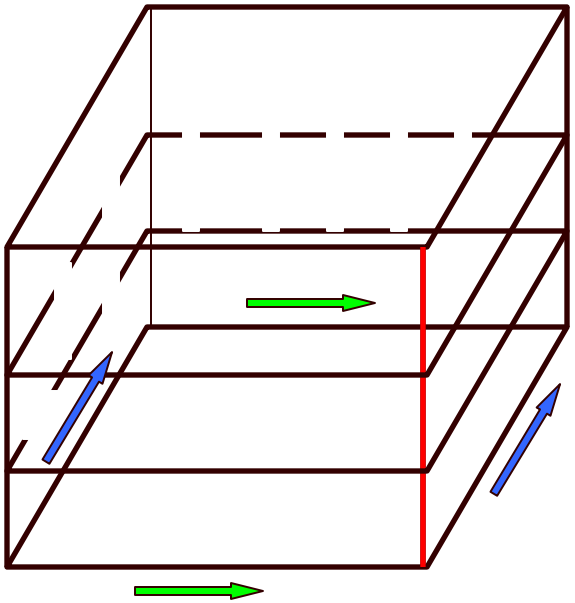
3-torus

- Is three torus a product?
- If so, what is it a product of?
- Explain in words and pictures – write a little paragraph with accompanying pictures.



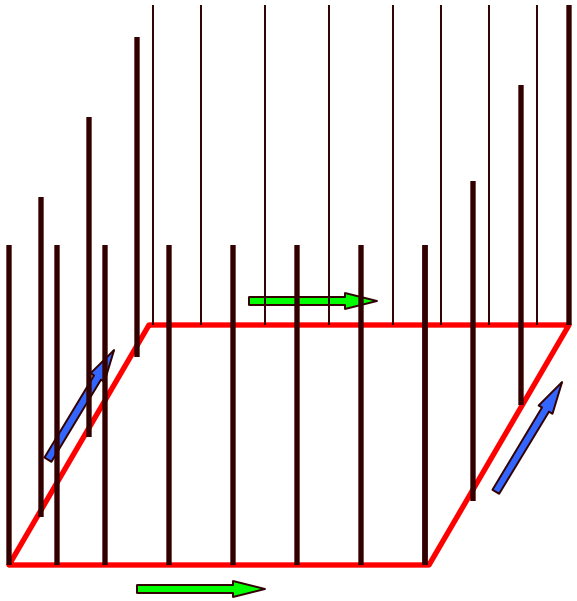
T^3

- Are top and bottom square surfaces?
- What about each square between the those two?
- What is the red segment?



T^3

Circle of tori



T^3

Torus of circles

$$T^3 = T^2 \times S^1$$

Flat 3-torus is a product of a flat 2-torus and a circle

Exercise

- Form a space by gluing the sides of a cube in the following manner:
  - Front to back with a side to side flip
  - Left to right normally
  - Top to bottom normally
- Of which spaces is this a product?