

Homework 9

Due on Wednesday, 4/12.

1. What is a hypersphere? We gave three different ways to think about hypersphere. Which one is your favorite and why? What are “horizontal” slices of a hypersphere?
2. What kind of geometry does a hypersphere admit? Do you know any other manifolds with the same geometry? “Yes” does not yield a full credit – give an example if you do.
3. What kind of geometry does a 3-torus admit? How would you have to glue the sides of a cube in order to get a 3-torus?
4. Take a cube and ‘glue’ the opposite sides in the following manner: the left side is glued to the right side normally; the top is glued to the bottom normally and front is glued to the back with a side to side flip. The manifold you get is all of the following:
 - a) K^2 bundle over a circle,
 - b) T^2 bundle over a circle,
 - c) a circle bundle over K^2 ,
 - d) a circle bundle over T^2 .

Explain why that is true. Draw pictures (don’t show all of it on one cube—draw 4 of them) and justify your answers.

Finally, is this manifold a product? If so, what is it a product of?