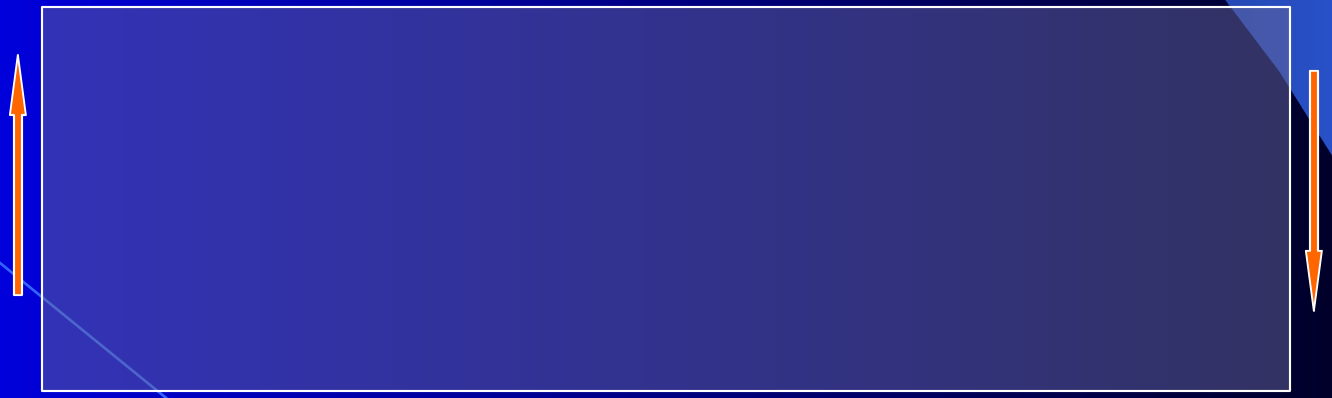
Vocabulary cont’d

Reminder

- Q: What is the difference between intrinsic and extrinsic topology?
- Q: A cylinder and a space obtained from it by cutting the cylinder and gluing it with a full twist of one end had the same intrinsic, but different extrinsic topology. How would you modify the gluing procedure to get two spaces with different intrinsic topology?

Möbius band



Reminder

- Local properties are those observable on a small region of your space.
- Global properties can only be observed by considering the whole space.

Exercise 1

Are the following discoveries global or local?

- The angles of a triangle are measured to be 62.2^{o}, 31.7^{o} and 89.3^{o}.
- An explorer set out to the west and returned from the east never deviating from a straight route.

Exercise 2

- Do torus and flat torus have the same:
  - Intrinsic global topology
    - yes
  - Extrinsic local geometry?
    - no

Exercise 3

- Do

and

have the same:

- Global extrinsic topology? • no
- Local intrinsic topology? • yes
- Local intrinsic geometry? • no

Manifolds

Definition: An n-dimensional manifold is a space that has the same local topology as the Euclidean n-dimensional space.

2 & 3 manifolds

- 2-manifold is a space with the local topology of a plane.
- 3-manifold is a space with the local topology of 3-space (Euclidean).

Question

- What is 1-manifold?
  - 1-manifold is a space with local topology of a line
    - If you take a little piece of your space it can be deformed to look like a little piece of the line

Different pieces of line

Different pieces of line

Different pieces of line

closed segment
(the endpoints are included)

open segment
(the endpoints are not included)

Definition revisited

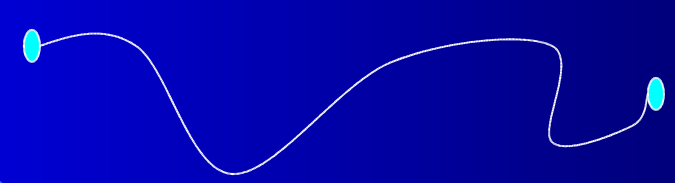
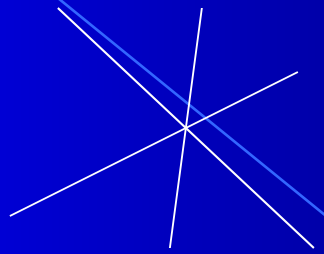
1-manifold is a space that locally has the same topology as an open segment

Or

A space is 1-manifold if its every point has a neighborhood with the topology of an open segment on the line.

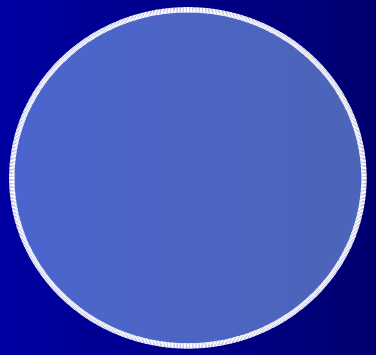
Exercise 4

Are these 1- manifolds?

- Line
  - yes
- Circle
  - yes
- 
  - no
- 
  - no

Question

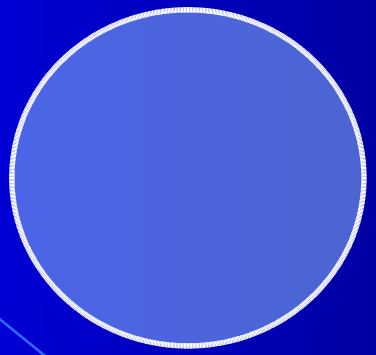
Is disk



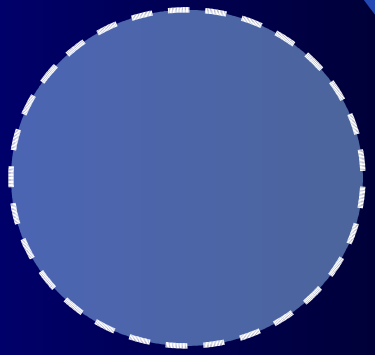
a manifold?

Remember: 2-manifold is a space with the local topology of a plane.

Neighborhood of a point in the plane



closed disk
(includes the
circular boundary)



open disk
(does not include
circular boundary)

Closed disk

A closed disk with center at P and radius is the set of all points in the plane whose distance from P is smaller than or equal to r.

$$D(P,r) = \{ Q \in E^2 \mid d(P,Q) \leq r \}$$

Open disk

An open disk with center at P and radius r is the set of all points in the plane whose distance from P is smaller than r.

$$B(P,r) = \{ Q \in E^2 \mid d(P,Q) < r \}$$

Circle

A circle with center at P and radius r is the set of all points in the plane whose distance from P is exactly r.

$$C(P,r) = \{ Q \in E^2 \mid d(P,Q) = r \}$$

Remark

- $B(P,r) \subset D(P,r)$ open disk is contained in the closed disk
- $C(P,r) \subset D(P,r)$ circle is contained in the closed disk
- $B(P,r) \cap C(P,r) = \varnothing$ open disk and circle have no point in common

Definition revisited

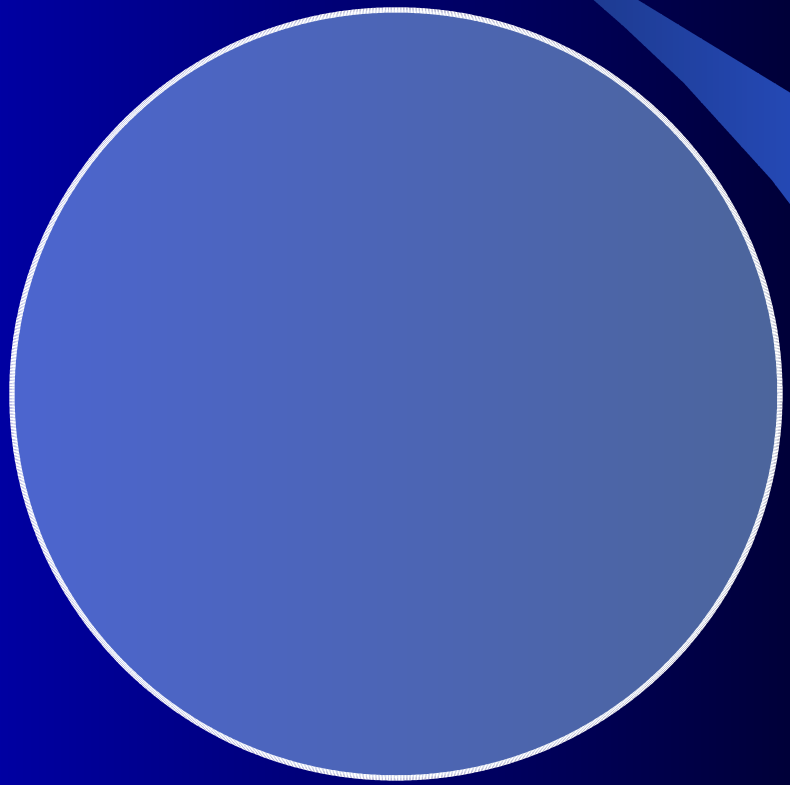
2-manifold is a space that locally has the same topology as an open disk.

Or

A space is 2-manifold if its every point has a neighborhood with the topology of an open disk in the plane.

Back to question

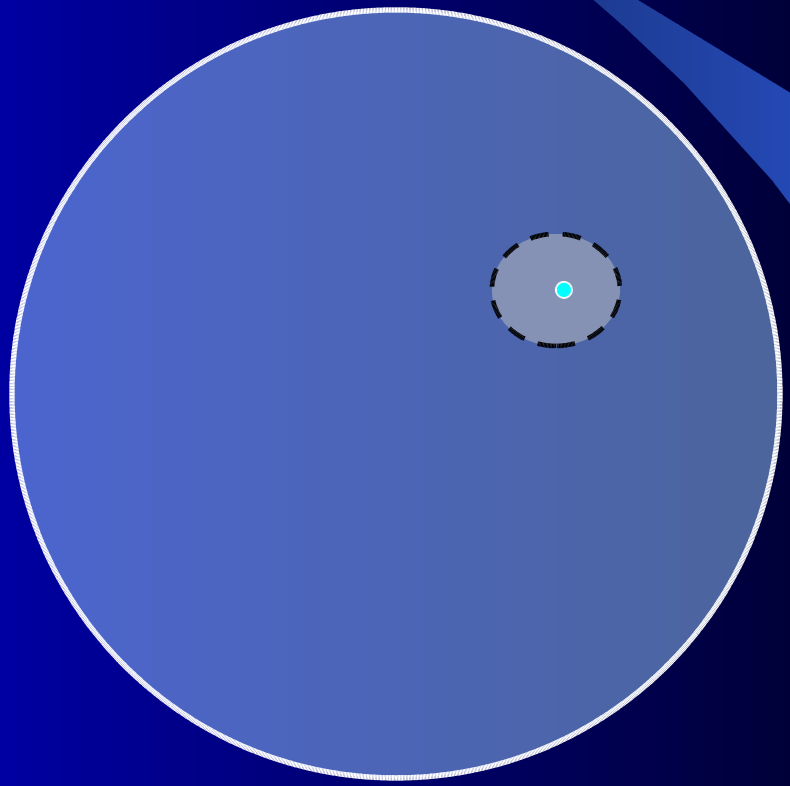
Is disk



a manifold?

Back to question

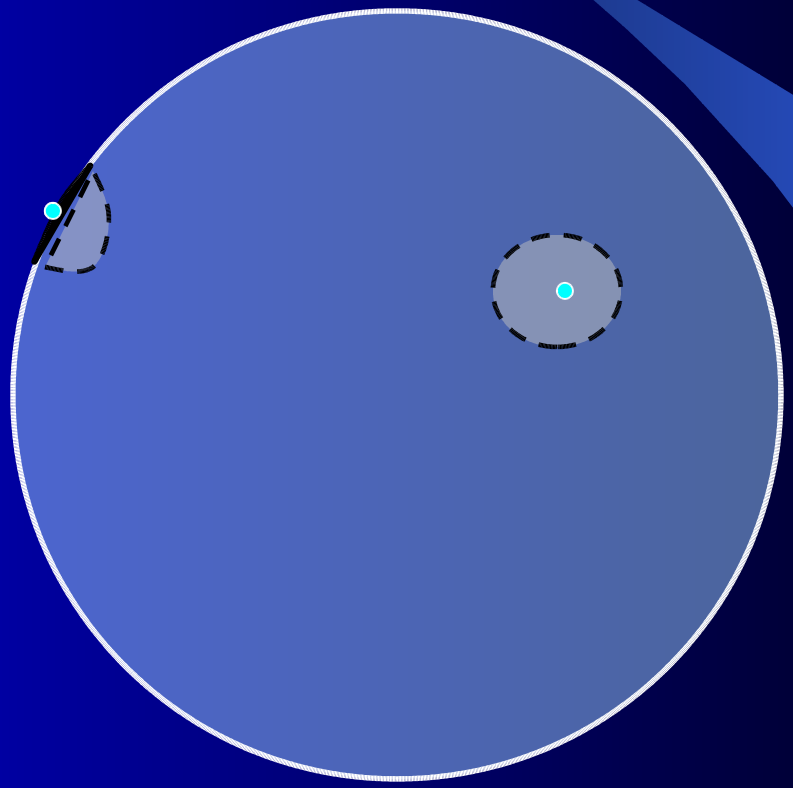
Is disk



a manifold?

Back to question

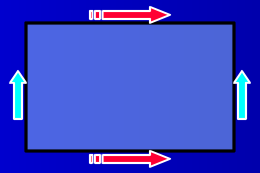
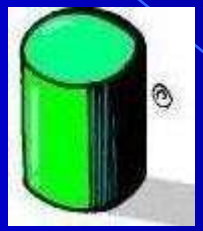
Is disk



a manifold?

Exercise 5

Which spaces are manifolds?

-  • no
- Infinite cylinder • yes
-  • yes
-  • yes
-  • no

Exercise

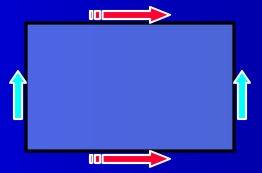
- How would you define a 3-manifold?
- What is the neighborhood of a point in E^3?
- What is the equivalent of an open disk, closed disk and circle in 3 dimensions?

Homogeneous manifolds

are those manifolds that have the same local geometry at all points.

Exercise 6

Which manifolds are homogeneous and which are not?

- infinite cylinder
  - yes
- 
  - yes
- 
  - yes
- 
  - no