Math 1220-5

February 27th, 2013

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Name: Key uNID:________ Points:______ Grade: ______

Quiz 6. 7.3, 7.4. **Indications:** This quiz is individual. Write all the steps showing your work. TOTAL: 20 pts. You have 20 min in order to complete the quiz

1. Compute the following integrals.

a) $\int \sin^2 t dt$. (3pts)

$\sin^2 t = \frac{1-\cos 2t}{2} = \frac{1}{2} - \frac{\cos 2t}{2}$

$= \int \frac{1}{2} - \frac{\cos 2t}{2} \, dt$

$= \frac{t}{2} - \frac{\sin 2t}{2 \cdot 2} + C = \frac{t}{2} - \frac{\sin 2t}{4} + C$

b) $\int \sin^5(4y)\cos^2(4y)dy$. (5pts)

$\int \sin(4y) \sin^4(4y) \cos^2(4y) dy.$

$= \int \sin(4y)(\sin^2(4y))^2 \cos^2(4y) d\cancel{y}$

$= \int \sin(4y)(1-\cos^2(4y))^2 \cos^2(4y) d\cancel{y}$

$u = \cos 4y \qquad du = -\frac{\sin 4y}{4} dy$

$= -4\int (1-u^2)^2 u^2 du$

$= -4\int (1-2u^2+u^4)u^2 du$

$= -4\int u^2 - 2u^4 + u^6 \, du.$

$= -4\left(\frac{u^3}{3} - \frac{2u^5}{5} + \frac{u^7}{7}\right) + C = -\frac{4}{3}\cos^3 \cancel{y} + \frac{8}{5}\cos^5 x - \frac{4}{7}\cos^7 x + C.$

c) $\int \sin(4t)\cos(5t)dt$. (3pts)

$\sin mx \cos nx = \frac{1}{2}(\sin(m+n)x + \sin(m-n)x)$

$= \frac{1}{2}\int \sin 9t + \sin(-t) \, dt$

$= -\frac{1}{2}\frac{\cos(9t)}{9} + \frac{1}{2}\cos t + C$

Remember $\sin(-t) = -\sin t$

and $\int \sin t \, dt = -\cos t + C$

d) $\int \frac{xdx}{\sqrt{3x+4}}dx$. (4pts)

$u = \sqrt{3x+4}$

$u^2 = 3x+4 \Rightarrow \frac{u^2-4}{3} = x \quad \Big| \quad \frac{2}{3}u\,du = dx.$

$$\int \frac{\frac{u^2-4}{3}\,\frac{2}{3}u\,du}{u}$$

$$= \frac{2}{9}\int \frac{(u^2-4)u}{u}\,du = \frac{2}{9}\int u^2-4\,du = \frac{2}{9}\frac{u^3}{3} - \frac{2}{9}4\cdot u + C.$$

$$= \frac{2}{27}\left(\sqrt{3x+4}\right)^3 - \frac{8}{9}\sqrt{3x+4} + C$$

e) $\int \frac{dt}{\sqrt{t^2+2t+5}}dt$. (5pts)

$t^2+2t+5+1-1$ Complete squares.

$(t+1)^2+4$

$$\int \frac{1}{\sqrt{(t+1)^2+4}}\,dt$$

$u = t+1$

$du = dt$

$$\int \frac{1}{\sqrt{u^2+4}}\,du$$

$u = 2\tan\theta$

$du = 2\sec\theta^2\,d\theta$

$\Rightarrow \frac{u}{2} = \tan\theta \Rightarrow \theta = \tan^{-1}(\frac{u}{2}).$

$$= \int \frac{\cancel{2}\sec^2\theta\,d\theta}{\cancel{2}\sec\theta} = \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$$

$$= \ln|\sec(\tan^{-1}(\tfrac{u}{2})) + \tan(\tan^{-1}(\tfrac{u}{2}))| + C$$

The final answer have to be in terms of "t"

You can stop here.

$$= \boxed{\ln|\sec(\tan^{-1}(\tfrac{\quad}{2})) + \tfrac{t+1}{2}| + C.}$$

$$= \ln\left|1 + \left(\frac{(t+1)}{2}\right)^2 + \frac{t+1}{2}\right| + C.$$

2. EXTRA CREDIT. Compute the integral.

$\int x^2 \cos x dx$. (5pts)

$$\boxed{\begin{array}{ll} u = x^2 & du = 2x\,dx \\ v = \sin x & dv = \cos x\,dx \end{array}}$$

$$= x^2\sin x - \int \sin x \cdot 2x\,dx.$$

$$= x^2\sin x - 2\int x\sin x\,dx$$

$u = x \quad du = dx$

$v = -\cos x \quad dv = \sin x\,dx$

$$= x^2\sin x - 2\left[-x\cos x - \int -\cos x\,dx\right]$$

$$= x^2\sin x + 2x\cos x - 2\int\cos x\,dx = \boxed{x^2\sin x + 2x\cos x - 2\sin x + C}$$