Math 1220-5

February 20th, 2013

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Name: Key uNID:_____ Points:_____ Grade: _____

Quiz 5. 7.1, 7.2. Indications: This quiz is individual. Write all the steps showing your work. TOTAL: 20 pts. You have 20 min in order to complete the quiz

1. Compute the following integrals.

a) $\int e^{at}\cos t dt$. The value **a** is a constant. (8pts)

By parts 1st time.

$u = \cos t \quad du = -\sin t\,dt$

$v = \frac{e^{at}}{a} \quad dv = e^{at}dt$

$= \frac{e^{at}}{a}\cos t + \frac{1}{a}\int e^{at}\sin t\,dt$

By parts 2do time.

$u = \sin t \quad du = \cos t\,dt$

$v = \frac{e^{at}}{a} \quad dv = e^{at}dt$

$= \frac{e^{at}}{a}\cos t + \frac{1}{a}\left[\frac{e^{at}}{a}\sin t - \frac{1}{a}\int e^{at}\cos t dt\right]$

$\therefore \int e^{at}\cos t dt = \frac{e^{at}}{a}\cos t + \frac{1}{a^2}e^{at}\sin t - \frac{1}{a^2}\int e^{at}\cos t dt$

$\int e^{at}\cos t dt + \frac{1}{a^2}\int e^{at}\cos t dt = \frac{e^{at}}{a}\cos t + \frac{1}{a^2}e^{at}\sin t + C.$

$\left(1+\frac{1}{a^2}\right)\int e^{at}\cos t dt = \frac{e^{at}}{a}\cos t + \frac{1}{a^2}e^{at}\sin t + C$

$\int e^{at}\cos t dt = \frac{\frac{1}{a}e^{at}\cos t + \frac{1}{a^2}e^{at}\sin t + C}{\left(1+\frac{1}{a^2}\right)}.$

$\int e^{at}\cos t dt = \frac{a^2\left(e^{at}\cos t + \frac{1}{a^2}e^{at}\sin t + C\right)}{a^2+1}$

b) $\int e^x \sec^2(e^x)dx$. (3pts) $u = e^x$, $du = e^x dx$

$\int \sec^2 u\,du = \tan u + C = \tan(e^x) + C.$

c) $\int \frac{\sin(\ln 4x^2)}{x} dx$. (4pts)

$u = \ln 4x^2 \qquad du = \frac{1}{4x^2} \cdot 8x\, dx$

$du = \frac{2}{x} dx$

$\frac{du}{2} = \frac{dx}{x}.$

$\frac{1}{2}\int \sin u\, du = -\frac{1}{2}\cos u + C$

$= -\frac{1}{2}\cos(\ln 4x^2) + C.$

d) $\int_1^e \sqrt{t}\ln t\, dt$. (5pts) ULIATE

$u = \ln t \qquad du = \frac{1}{t} dt$

$v = \frac{2t^{3/2}}{3} \qquad dv = \sqrt{t}\, dt$

$dv = t^{1/2} dt$

$\int dv = \int t^{1/2} dt$

$= \frac{2t^{3/2}}{3}\ln t - \int \frac{2t^{3/2}}{3}\cdot\frac{1}{t} dt$

$= \frac{2t^{3/2}}{3}\ln t - \frac{2}{3}\int t^{1/2} dt$

$= \frac{2}{3}t^{3/2}\ln t - \frac{2\cdot 2\cdot t^{3/2}}{3\cdot 3} + C = \frac{2}{3}t^{3/2}\ln t - \frac{4}{9}t^{3/2}\Big|_1^e$

$\left[\frac{2}{3}e^{3/2}\ln e - \frac{4}{9}e^{3/2}\right] - \left[\frac{2}{3}1^{3/2}\ln 1 - \frac{4}{9}\cdot 1^{3/2}\right]$ ($\ln 1 = 0$)

$= \frac{2}{3}e^{3/2} - \frac{4}{9}e^{3/2} + \frac{4}{9}.$

2. EXTRA CREDIT. Compute the integral.

$\int \frac{\sec^3 x + e^{\sin x}}{\sec x} dx$. (5pts)

$= \int \frac{\sec^3 x}{\sec x} dx + \int \frac{e^{\sin x}}{\sec x} dx$ $\longrightarrow$ $u = \sin x$, $du = \cos x\, dx$

$= \int \sec^2 x\, dx + \int \cos x\, e^{\sin x} dx.$

$= \tan x + \int e^u du = \tan x + e^u + C$

$= \tan x + e^{\sin x} + C.$ //