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Research Statement

I work in various fields of applied mathematics, in particular, inverse and ill-posed problems, inhomogeneous media, optimization and variational methods, wave propagation, mathematical modeling. I work with mathematical and computational aspects of nonlinear inverse problems for partial differential equations and imaging, which often can be formulated as optimization, control, or variational problems. These problems are ill-posed and computationally intensive, many of them have practical and industrial applications in geophysical and medical imaging, in remote sensing and material science, in nondestructive testing and biomaterials. More specifically, I dealt in past and continue to work at present on the following problems and mathematical techniques:

– Nonlinear problem of identification of coefficients of partial differential equations or the source terms, such as identification of functions describing the properties of the medium (for instance, thermal or electric conductivity, density, complex permittivity) inside a domain from boundary measurements such as Neumann-to-Dirichlet map.

– Optimization of the eigenvalues of an operator, eigenvalues in the boundary condition, minmax problems, – these techniques generated novel approaches in inverse and structural design problems, such as focusing of boundary sources and robust optimal design.

– Inverse homogenization and inhomogeneous materials where spectral representation and analyticity of the effective properties of a random mixture yield to a novel inverse problem for the microstructure of the mixture. Together with M.Y.Ou (U. Central Florida) I recently started to work on extension of inverse homogenization approach to visco-elastic materials and fluid/solid composites having in mind application to osteoporosis for evaluation of the bone density and structure from ultrasound measurements.

– Non-quadratic and nonsmoothing regularization of ill-posed problems, such as total variation penalization and regularization based on nonnegativity constraint. I dealt with a spectrum of inverse problems: in potential theory, computer and electric tomography, inverse electromagnetic problems. My current interests are in multiscale electromagnetic imaging of dispersive media, when the complex permittivity reconstructed on a large scale inversion step, is utilized on the microscale step to recover information about the structure of a random composite material using spectral representation of the effective properties. Inverse problems I am working on, have numerous applications in geophysics, that is why my research in this area and my students were funded by Schlumberger Doll Research.

– Nonlinear materials: wave propagation and dissipation, phase transition in bi-stable materials and nonlinear random networks; I collaborate with Andrej Cherkaev and Leonid Slepyan on a project funded by ARO. I also work on homogenization for soft biological materials modeled as a Cosserat continuum, in particular for modeling blood clots (collaboration with Aaron Fogelson, Jim Keener, and Carlos Bonifasi-Lista (a Ph.D. student)).

– I use a variety of numerical methods, and I published two papers in mathematical game theory about measures generated by set functions.

– I am also collaborating on a math undergraduate education NSF-CCLI funded project, I am a co-PI together with Robert Palais and Andrej Cherkaev. Together with Andrej we are working on a textbook on calculus of variations and applications ([47]). Finally, in spare time I collaborate with Andrej on recreational math - we collected math jokes and put them on the internet, it became one of the most popular webpages – the number of visitors exceeded 150,000 since we launched it several years ago. The web address is www.math.utah.edu/∼cherk/mathjokes.html

1 Ill-posed and inverse problems

1.1 Reconstruction of the microstructure of a mixture

1. **Inverse homogenization** theory and **inverse problems for a medium with microstructure**. The homogenization theory developed efficient approaches, which allow to derive equations for averaged fields and characterize effective properties of periodic or random mixtures of several phases. These results have immediate applications to composite materials and materials with microstructure. Materials with microstructure are everywhere around us: Bones, lungs, and blood clots, sea ice and artificial composites, geological minerals and oil bearing rocks are particular examples.

In many practical problems, the effective properties of a composite medium can be measured and the unknown structure of the material is of real interest. For instance, it is important to be able to estimate porosity of a fractured geological formation or morphology of biological tissues from their response to the applied electromagnetic field. When the scale of the structure of a random mixture of two materials is much smaller than the wavelength of the applied electromagnetic signal, only homogenized or effective response of the structure is present in the measurements. However, I proved that it is still possible to reconstruct information about the microstructure using effective measurements in a range of frequencies. I formulate an inverse homogenization problem as a problem of extracting information about the microstructure from effective or homogenized measurements. This structural information is contained in the spectral measure μ in the Stieltjes representation of the effective complex permittivity. The spectral measure can be reconstructed from effective measurements and used to characterize parameters of the microstructure or to estimate other effective properties of the same material.

If we consider a stationary random fine-scale mixture of two materials with properties ϵ_1 and ϵ_2, and introduce a characteristic function χ of the region $\mathcal{O}_1$ occupied by the first material for a realization $\eta \in \Omega$, where Ω is the set of all realizations of the random medium,

$$\chi(x,\eta) = \begin{cases} 1, & x \in \mathcal{O}_1, \\ 0, & \text{otherwise} \end{cases}$$

the complex permittivity of the medium is modeled by a (spatially) stationary random field $\epsilon(x,\eta)$, $x \in \mathbf{R}^d$ and $\eta \in \Omega$, $\epsilon(x,\eta) = \epsilon_1\chi(x,\eta) + \epsilon_2(1-\chi(x,\eta))$. Suppose that two different fields are applied in this random medium, the electric field E_ϵ and the temperature gradient field E_γ. The stationary random fields $E_\sigma(x,\eta)$ and $J_\sigma(x,\eta)$, $\sigma = \epsilon, \gamma$, are related by $J_\sigma(x,\eta) = \sigma(x,\eta)E_\sigma(x,\eta)$ and satisfy the equations

$$\nabla \cdot J_\sigma = 0, \qquad \nabla \times E_\sigma = 0, \qquad \langle E_\sigma(x,\eta)\rangle = e_k, \qquad \sigma = \epsilon, \gamma. \tag{1}$$

Here e_k is a unit vector in the k^{th} direction, for some $k = 1, \ldots, d$, and $\langle \cdot \rangle$ means ensemble average over Ω or spatial average over all of $\mathbf{R}^d$. The effective tensors ϵ^* and γ^* are defined as coefficients of proportionality between the averaged fields: $\langle J_\epsilon\rangle = \epsilon^*\langle E_\epsilon\rangle$ and $\langle J_\gamma\rangle = \gamma^*\langle E_\gamma\rangle$. We notice that both problems are related by the same function χ:

$$\nabla \cdot (\epsilon_1\chi(x,\eta) + \epsilon_2(1-\chi(x,\eta)))\, E_\epsilon = 0, \qquad \epsilon^* = \langle \epsilon E_\epsilon\rangle \tag{2}$$

and

$$\nabla \cdot (\gamma_1\chi(x,\eta) + \gamma_2(1-\chi(x,\eta)))\, E_\gamma = 0 \qquad \gamma^* = \langle \gamma E_\gamma\rangle. \tag{3}$$

Two problems I would like to solve are the following:

- Suppose that the effective complex permittivity ϵ^* is measured. Can we characterize the function χ which defines the structure of the mixture?

- If the effective property ϵ^* is measured, can we find the other effective property γ^*?

The idea of the approach I developed is based on an analytic representation of various effective properties of a composite through the spectral measure μ in the Stieltjes analytic integral representation of the effective complex permittivity ϵ^*. This analytic integral representation was developed by Bergman, Milton, and Golden and Papanicolaou in the course of computing bounds for the effective permittivity ϵ^* of an arbitrary two-component mixture. Introducing $s = 1/(1-h), h = \sigma_1/\sigma_2$, the equations (2) and (3) can be written as

$$\nabla \cdot \chi E = s \nabla \cdot\ E\ , \qquad s = \frac{1}{1-h}\ , \qquad h = \sigma_1/\sigma_2\ , \qquad \sigma = \epsilon, \gamma\ . \tag{4}$$

Let $\nabla\phi$ be a perturbation of the constant field e_k , so that $E = e_k + \nabla\phi$. Then, $\nabla \cdot \chi\,(\nabla\phi + e_k) = s\,\Delta\phi$. Introducing an operator $\Gamma = \nabla(-\Delta)^{-1}(\nabla\cdot)$, we can express E as a function of $\Gamma\chi$, $E = s(sI + \Gamma\chi)^{-1}e_k$. The spectral resolution of $\Gamma\chi$ with the measure Q results in the spectral representation for the field E, which is used to obtain the analytic representation for the effective property. Indeed, the function $F(s) = 1 - \sigma^*(s)/\sigma_2 = \langle\, s^{-1}\,\chi\, E,\, e_k\,\rangle$, can be represented as

$$F(s) = \langle\, \chi\,(\,sI + \Gamma\,\chi\,)^{-1} e_k, e_k \rangle = \int_0^1 \frac{\langle\, \chi\, dQ(z)\, e_k,\, e_k \rangle}{s-z} \tag{5}$$

Let μ be a positive function of bounded variation, corresponding to the spectral measure Q, $\;d\mu(z) = \langle\, \chi\, dQ(z) e_k, e_k \rangle$, then

$$F_\sigma(s) = 1 - \frac{\sigma^*}{\sigma_2} = \int_0^1 \frac{d\mu(z)}{s-z}\ , \qquad s = \frac{1}{1 - \sigma_1/\sigma_2}\ , \qquad \sigma = \epsilon, \gamma\ . \tag{6}$$

The spectral function μ contains all information about the function χ and about the structure of the medium. Hence, if we can reconstruct the measure μ from the effective measurements, we reconstruct information about the function χ. I show in [39] that the measure μ can be uniquely recovered from the effective complex permittivity given on an arc $\mathcal{C} \subset C$ in the complex plane: $\epsilon^*(s), s \in \mathcal{C}$. I also introduce a concept of $S-$*equivalency* for micro-geometries of mixtures which cannot be distinguished by effective measurements, and show that the $S-$not-equivalent structures correspond to different spectral functions. I continue to work on the questions 'What geometries can be uniquely characterized from effective data? and What classes of geometric structures admit unique characterization from effective data?'

Regularization The inverse problem of reconstruction of the spectral measure μ can be reduced to an inverse potential problem. Indeed, the function $F(s)$ admits a representation as a logarithmic potential of the measure μ

$$F(s) = \frac{\partial}{\partial s} \int \ln|s-z|\ d\mu(z), \qquad \partial/\partial s = (\partial/\partial x - i\ \partial/\partial y) \tag{7}$$

The reconstruction problem for the logarithmic potential is exponentially ill-posed and requires regularization. The potential function u is a solution to the Poisson equation

$$-\,\Delta u = \psi, \qquad \mathrm{supp}(\psi) \subset \Omega, \tag{8}$$

where ψ is the density of the mass distribution in Ω. Solution of the problem is given by the Newtonian potential with $d\,\mu(z) = \psi\, dz, \;\; z \in \Omega$. The inverse problem is to find ψ given values of $\partial u\,/\partial n$, or ∇u. To construct the solution we formulate the minimization problem:

$$||A\mu - F|| \to \min_{\mu \in \mathcal{M}}, \tag{9}$$

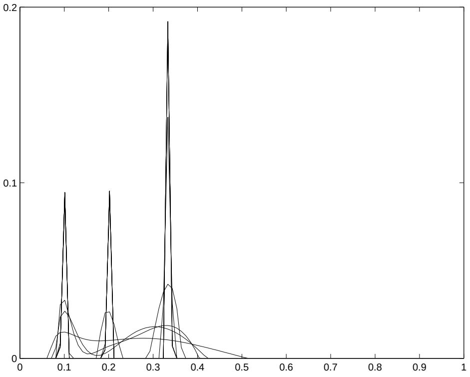

Figure 1: Reconstruction of a three-resonance spectral function μ using quadratic, total variation, and nonnegativity constraints. The quadratic regularization gives oversmoothed solutions. The total variation penalization leads to better reconstruction results. The numerical algorithm based on the non-negatively constrained minimization accurately recovers the three-resonance function.

where A is the operator (7). The regularization algorithm is based on constrained minimization with a quadratic or a total variation stabilization functional. Another regularization approach uses a **nonnegativity constraint** for the set of minimizers. This is a very promising direction since this constraint describes the true physical property, it does not depend on voluntarily chosen parameters of the inversion scheme as in the case with the constraint for a norm of the solution; in the last case, the constructed solution depends on the value of the constraint, which is unknown a priori. This regularization approach constructs the inversion algorithm which works almost as an "ideal resolution" operator: The algorithm is able to practically exactly recover three delta functions shown in Figure 1. Indeed, the true and the reconstructed solutions are indistinguishable on this scale.

Spectral coupling of various properties of the mixture The obtained in (6) representation is used to answer the second question. Various properties (for instance, thermal, electric, and hydraulic conductivity) of the same mixture are coupled through the function μ in their integral representation [39,40]. This explains coupling between different properties of the composite material, - the fact widely used in practice in empirical or semi-empirical relations such as Kozeny-Carman or Katz-Tompson relations. Indeed,

$$\epsilon^*(s) = \epsilon_2 - \epsilon_2 \int_0^1 \frac{d\mu(z)}{s-z}\,, \qquad s = \frac{1}{1-\epsilon_1/\epsilon_2} \tag{10}$$

$$\gamma^*(s') = \gamma_2 - \gamma_2 \int_0^1 \frac{d\mu(z)}{s'-z}\,, \qquad s' = \frac{1}{1-\gamma_1/\gamma_2} \tag{11}$$

and if the function μ is known from the measurements of ϵ^*, evaluation of the effective thermal conductivity γ^* reduces to a simple calculation. Together with my Ph.D. student, Dali Zhang, we use data of the complex permittivity of various geophysical materials in a frequency interval to find the function μ and to evaluate the effective thermal conductivity of the same composite [41].

Bounds for volume fractions Characterization of the microstructural parameters of the mixture is another application of the representation (6). In collaboration with Alan Tripp and Jeff Hulen (in [14, 18]), and Kenneth Golden (in [25]), I developed the bounds for the volume fractions of materials in anisotropic and isotropic mixtures. Application of the method to estimating the brine volume in bubbly sea ice shows a very good agreement with experimental data [25].

Current work and future plans I continue to work on several aspects of the problem:

1. I plan to extend the previously obtained results and numerical algorithms to anisotropic mixtures and materials with interface between the phases. The goal is to characterize classes of microgeometries uniquely recovered from effective properties.

2. I work on inverse homogenization and reconstruction of the microstructural parameters for visco-elastic composites. This has an application to imaging the structure of bones and blood clots. The goal is to develop a model incorporating microstructural parameters, and then to formulate and solve the inverse problem of evaluating morphology of the clot and structure and density of the bone using ultrasound measurements.

3. The requirements of uniqueness of the reconstruction of the spectral measure μ suggest an approach to two-scale inverse electromagnetic problem. This problem couples the spectral function and the complex permittivity in a range of frequency reconstructed on a large scale step. This forms an approach to two-scale inverse problem: The first step is a large-scale time (or frequency)-domain problem for dispersive materials, the second step is the inverse problem for the spectral measure, and then recovering the microstructural information.

1.2 Electric and electromagnetic tomography and focusing of boundary sources

The problem is to reconstruct the spatially varying coefficients of a partial differential equation inside the domain from the Neumann to Dirichlet data on the boundary. Problems of this type arise in geophysical surface and borehole electromagnetic imaging such as monitoring of conducting contaminant plumes or flooding oil fields, as well as in engineering and biomedical applications. Physically, the problem is to reconstruct the earth's properties σ and ϵ from measurements of the electric and/or magnetic fields generated in the medium by applied electromagnetic sources. I developed an efficient computational approach based on focusing of optimal boundary electric currents introduced by Isaacson and Cheney. The optimized boundary sources which are given by the dominant eigenfunctions of the Neumann-to-Dirichlet map, generate the optimal finite dimensional approximation of the infinite dimensional inverse problem. I show that the eigenfunctions of the Neumann-to-Dirichlet map focus the currents inside the domain and maximize the energy of the scattering current over the region of the inclusion, maximizing the resolution of the conductivity problem. The numerical algorithm based on focusing of the boundary sources becomes especially effective because it allows to capture the most of information about the solution and leads to data compression.

It was shown in works of Kohn and Vogelius; Uhlmann and Sylvester; Nachman; and very recently by Astala and Paivarinta, that the Dirichlet-to-Neumann map $u \to \Lambda_\sigma u$ on the boundary uniquely determines the coefficient function σ of the conductivity equation $L_\sigma u = 0$. I consider the inverse problem as a problem of minimization of the norm of the difference between the given Neumann-to-Dirichlet map $R_{\bar{\sigma}} f$ (measured data) and the simulated map $R_\sigma f$, where f is the injected current on the boundary, $R = \Lambda^{-1}$:

$$||R_{\bar{\sigma}} f - R_\sigma f|| \to \min_\sigma$$

The eigenvalues of this boundary map $f \to (R_{\bar{\sigma}} - R_\sigma) f$ decay very fast if the medium is conducting, therefore only data due to several dominant eigenfunctions are of importance for the reconstruction procedure. An algorithm which iteratively uses only data due to dominant eigenfunctions gives rapid and accurate imaging results.

I extend this approach to the low frequency inverse electromagnetic problem modeled by Maxwell's equations at fixed frequency. Since the operator is not self-adjoint, the iterative minimization requires computation of the solution of the adjoint problem on each iteration step. To avoid this, I introduce a J-self-adjoint extension of the Maxwell's operator and show that the inverse problem can be formulated

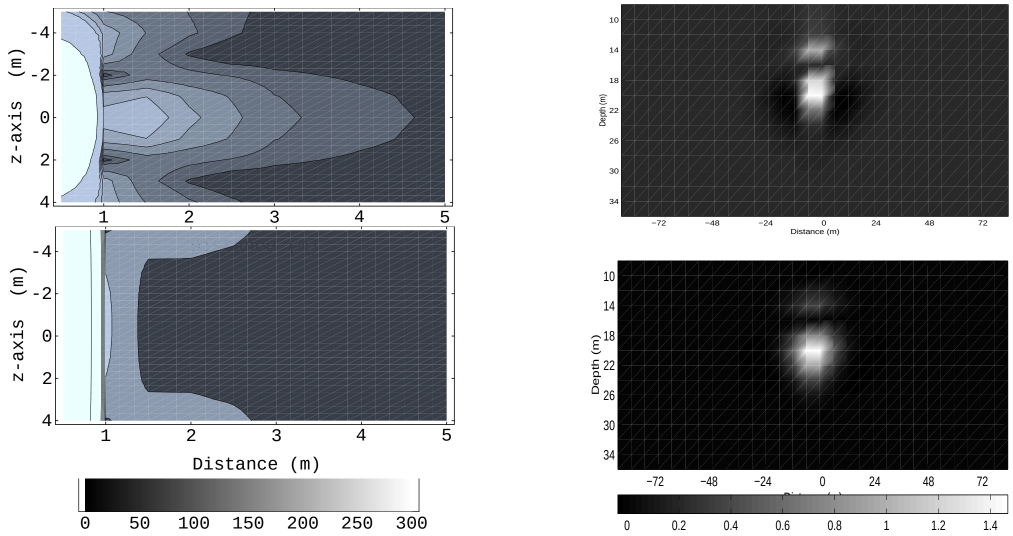

Figure 2: Left figure: Distribution of currents inside the earth due to a focused array of magnetic dipoles (top) and due to a conventional array (bottom). Right figure: Reconstruction of the complex permittivity (top) and the conductivity (bottom) using one iteration of focused array imaging.

as a problem for a self-adjoint operator; this leads to efficient computational method. Figure 2 (left) shows results of optimization of electromagnetic inductive sources, magnetic dipoles, in a geophysical problem of detection of a resistive layer embedded in a conductive medium, when the magnetic dipoles are located in the vertical borehole on the left. The top left figure shows the field due to the first eigenfunction: In spite of the layer outside the borehole being resistive, the field is concentrating there, meanwhile the field generated by the conventional current of uniform intensity avoids the resistive layer (bottom figure). Right figure shows results of reconstruction of the complex permittivity ϵ (top) and the conductivity σ (bottom) using one iteration of focused array imaging for a model of an inclusion of different conductivity and complex permittivity embedded in a halfspace.

Current work: I work on the time domain inverse electromagnetic problem for dispersive media. This is a "large scale" part of the two-step inversion for a medium with a microstructure. The memory kernel reconstructed on this step provides data for the microscale inverse problem of recovering the spectral function μ.

1.3 Regularization in inverse problems

A characteristic feature of inverse problems is their ill-posedness. I worked with and developed various regularization techniques for ill-posed inverse problems: Quadratic penalization in L_2, H_1, W_2^2, non-smoothing total variation regularization which imposes constraint on variation of the minimizer in the domain, regularization using non-negativity constraints.

I consider as an example inverse potential problem (8) which also solves an important problem in geophysics: Inverse gravity problem [38]. Regularized algorithm for this exponentially ill-posed problem can be formulated as a penalized least squares minimization problem,

$$\min P^{\alpha}(\psi) = \min_{\psi} \left(\|K\psi - g\|_2^2 + \alpha\, J(\psi) \right).$$

Here g is the measured values of the vertical derivative of u on $\partial\Omega$, K is the vertical derivative of the

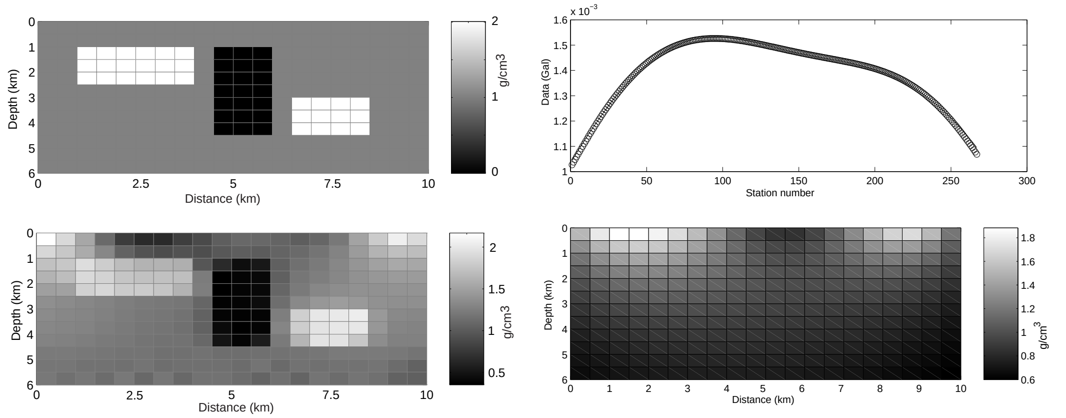

Figure 3: Example of regularization using different stabilization functionals: True density function for the fault model (top left figure) and results of reconstruction. Right figure: Reconstruction using a smoothing regularization scheme together with the observed and predicted data. Left bottom figure: Density function reconstructed using total variation stabilization functional. [38]

Poisson integral, $K : \psi|_\Omega \to n \cdot \nabla(-\Delta)^{-1}\psi|_{\partial\Omega}$. The stabilization functional J imposes constraints on the norm of the solution: $J(\psi) \leq C$. I also used this approach to stabilize the Galerkin method of solving the boundary value problem with an oblique derivative using a non-orthogonal system of basis functions on irregular grids ([4,9]).

Total variation regularization for inverse problems with nonsmooth properties Together with my Ph.D. student Hugo Bertete-Aguirre (Ph.D. '98, currently in Livermore National Lab), and in collaboration with Michael Oristaglio and with Aaron Fogelson, we developed a nonsmoothing regularization technique [38,45] applicable to geophysical imaging. To prevent penalization of sharp changes of the solution in the minimization problem, the method uses the variation or BV-norm of the solution as a constraint for the set of minimizers. Unlike in image segmentation problems where the total variation functional has been used previously, the image itself is a solution of an exponentially ill-posed inverse potential problem. Analysis of the resulting Euler-Lagrange equation

$$(K^*K\psi - K^*g) + \alpha\nabla \cdot \frac{\nabla\psi}{\sqrt{|\nabla\psi|^2 + \beta}} = 0$$

with β being a small parameter to avoid singularity at zero, shows that in the course of iterations, the solution is driven to a blocky structure. Our work shows that the method can be effectively used for reconstruction of sharp contrast features of the solution. These results are very promising for geophysical and other industrial applications.

2 Robust structural design

Extension of my work on optimization of the eigenfunctions of the Neumann-to-Dirichlet map, is a robust approach to optimal design (collaboration with Andrej Cherkaev). We formulated a new problem of robust structural design of elastic structures for an uncertain loading as minimization of the maximum of the energy $\mathcal{J}$ stored in the domain. An integral constraint for the set of admissible forces $\mathcal{F}$ is assumed to be given. We introduce an integral characteristic of the domain similar to the center of masses or principal eigenfrequency - the principal compliance Λ of the domain equal to the maximum of the stored energy over all admissible forces,

$$\Lambda = \max_{f \in \mathcal{F}} \mathcal{J}(C, f).$$

The principal compliance Λ is the maximal compliance under the extreme, worst possible force f. We show ([29,42]) that the problem for the extreme force reduces to an elasticity problem with mixed nonlinear boundary condition, which may have multiple solutions. If the L^2norm of the forces is constrained (with ψ being the weight function),

$$\mathcal{F} = \{f : \int_{\partial\Omega} f^T \psi f \leq 1\},$$

the principal compliance Λ equals the reciprocal of the Steklov eigenvalue, which is an eigenvalue in the boundary condition.

The robust optimal design problem is formulated as minimization of Λ with respect to distribution of the stiffness C in the domain, this leads to a min-max problem for the energy stored in the structure:

$$\min_{C} \max_{f \in \mathcal{F}} \mathcal{J}(C, f)$$

The maximum of the energy is chosen over the constrained class of loadings, while the minimum is taken over the design structure parameters. Generally, the functional $\mathcal{J}$ is not a saddle-point functional, hence the existence of a unique solution cannot be guaranteed. The optimization with respect to the designed structure C accounts for possible multiplicity of extreme loadings. When there are two materials that are to be placed over the domain to create an optimal structure, the problem is a non-convex variational problem, and mixtures or composite materials are natural candidates for the solution. We reformulate the problem extending the class of structures to admit materials with fast oscillating properties. The type of the solution depends on the constraint. Continuous change of the loading constraint causes bifurcation of the solution of the optimization problem. We show ([42]) that an invariance of the set of constraints under a symmetry transformation leads to a symmetry of the optimal design. We also apply this approach to biomaterials, which 'are designed' to work under not known in advance loadings.

3 Nonlinear problems

3.1 Nonlinearity in the boundary condition

The inverse gravity problem described above stems from a more general problem I dealt with in my previous research. It arises in studies of the geomagnetic field, and is in many aspects similar to the problem of mathematical geodesy. Physically, the problem is to reconstruct components of the vector of the geomagnetic field ∇u from measurements of the modulus of the vector. This reduces to differentiation of the solution u of the non-linear boundary problem:

$$\Delta u = 0 \ \text{ in } \Omega, \quad |\nabla u| = F \ \text{ on } \partial\Omega.$$

The problem was formulated by Backus, 1970, who brought an example showing that the solution is non-unique. I show in [4,11], that the non-uniqueness is caused by the non-uniqueness of the solution of the linearized problem, which is a non-classical problem with oblique derivative violating the Fredholm alternative. The linearized problem is

$$\Delta u_1 = 0 \quad \text{in} \;\; \Omega \; , \quad \frac{\partial u_1}{\partial \mathbf{t}} = f \quad \text{on} \;\; \partial\Omega \; , \quad \mathbf{t} = \frac{\nabla u_0}{|\nabla u_0|},$$

where $u = u_0 + u_1$, with u_0 being known, $f = F - |\nabla u_0|$. Due to the dipole character of the geomagnetic field, there is a line M on the boundary $\partial\Omega$ (called magnetic equator) where the vector of differentiation $\mathbf{t}$ in the boundary condition is tangent to the boundary surface. This results in violation of the condition for ellipticity; the problem is non-Fredholm, which leads to the non-uniqueness of the solution. From results of Mazia and Paneach, it follows that the solution is defined up to an additional boundary condition: the values of the function u_1 on M should be assigned to ensure uniqueness: $u_1|_M = \phi$. This allowed me to *a priori* estimate the uncertainty of the solution at points on the boundary surface $\partial\Omega$, which depends on the distance to M. This theory explains the long-standing geophysical problem of "false magnetic anomalies" near the equator.

Numerical scheme for this non-linear boundary value problem is based on asymptotic expansions [3,4]. Several small parameters account for the curvature of the earth's surface, the deviation of the differentiation vector $\mathbf{t}$, and the linearization of the problem near a known approximate solution. The coefficients u_l^k of the asymptotic expansion are harmonic functions obtained from a sequence of linear boundary problems with constant coefficients. The estimates of the solution are obtained for the non-linear problem formulated for the case when the norm $||u||_M$ on M is negligible. I show that the solution is unique and the expansion asymptotically converges to the exact solution. The problem is reduced to computation of derivatives of a 3D harmonic function from the values of the oblique derivative given on a 2D manifold. This is an ill-posed problem. The regularized solution is constructed with a three-dimensional pseudodifferential operator of Hilbert transform type.

3.2 Wave propagation and phase transition in nonlinear networks

In collaboration with Leonid Slepyan and Andrej Cherkaev, I work on modeling of propagation of waves and phase transition in nonlinear materials. We assume a nonlinear strain-stress characteristic of links in a one- or two-dimensional network; this constitutive relation is given by a function with two stable branches and a gap or unstable interval between them. Based on an approach developed by Slepyan for modeling fractures, we were able to analytically describe the wave propagation in such bi-stable piece-wise linear materials and calculate the velocity of the wave of phase transition as well as the dissipation of the energy associated with the transition to the second stable phase. The approach is based on separation of singularities corresponding to different waves in the complex plane and solving a nonlinear equation for the speed of the phase transition wave, which is formulated using an assumption that the speed of the phase transition is constant. Obtained results relate to cellular materials; developed numerical algorithms permit computation of nonlinear waves and waves of phase transition in materials with more complicated constitutive relations. Numerical simulations support the analytically obtained results and demonstrate that materials characterized by bistable diagrams are capable of consuming large amounts of energy due to partial damage during the transition to the second stable state. The increase of the energy absorbed by the material is also achieved by delocalization of the damage process. Optimizing parameters of the stress-strain constitutive relation, we construct a material in which the damage is distributed throughout the body instead of being localized in a particular necking process. This is current work funded by ARO, and is aimed at creation of new cellular materials capable of sustaining much higher level of damage comparing to conventional materials. The methods used are wave propagation, analyticity of the solution, causality principle, optimization of random structures.

Math Teaching

Teaching is rewarding. I am always excited about the subject I am teaching. I like to present ideas behind mathematical proofs, to show connections and relations between mathematical objects, I like to demonstrate proofs which emphasize beauty and elegance. I am sure that we all became mathematicians because at some point in our lives we have delighted in this beauty. I am glad when my students can see and appreciate it.

When I teach graduate courses, I usually supplement the textbook by notes and additional material from other books; I give projects related to the course material and encourage students to suggest their own topics related to their research area. This especially concerns to my Nonlinear Dynamics and Chaos course where fractals and chaos give many fun topics for a project, and some of the students write their own animation programs to present in class. I supervise graduate and undergraduate (REU) students. My former Ph.D. student Hugo Bertete-Aguirre (Ph.D. 1998) is working in Livermore National Lab. For his thesis, Hugo worked on total variation regularization of ill-posed geophysical problems; later he worked with me as an industrial postdoc: He was supported by a grant from Schlumberger Doll Research. Another student Altaf Khan (M.S., 2000) is currently working on his Ph.D. in Mechanical Engineering Dept. in Utah. For his master project, Altaf was working with me on a computer tomography problem. At present, I supervise two Ph.D. students: Dali Zhang, with whom we work on multiscale electromagnetic imaging, and Carlos Bonifasi-Lista who works on blood clots modeling and monitoring. I usually expose my graduate students to different ideas, discuss a variety of ways to proceed, and let them choose what seems to be most interesting and exciting for them.

In undergraduate classes, I try to give many examples of mathematical logic. Sometimes I show a problem as a puzzle, or explain that math proofs not only develop logical thinking, but also show why mathematical objects are the way they are. I sometimes deal with hard cases: Once I had a student in Calculus I who in the beginning of the semester burst into tears when I talked with her after class because she hated and feared math. By the end of the class her own perception of her mathematical abilities had changed.

I love watching students start to see math as a set of intriguing puzzles and no longer fear it.

I graduated from a special mathematical high school for math geeks attached to Leningrad University. That was the best mathematical school in the country, possibly the best in the world. For two years, we thought and talked about math all day and sometimes all night, - it was a boarding school. We were taught an extraordinary curriculum, and we were fascinated by math and its enigmas. Now I am trying to bring this excitement in the classroom to share it with my students. One of them after taking two courses from me, wrote in a class evaluation: ”this class has left me with not only a better understanding of math but a love for learning math...”