1. Two equal masses $m$ are connected by three springs with spring constants $c_1 = 1$, $c_2 = 1$, $c_3 = 2$. Spring 1 is fixed on the top, and spring 3 is fixed at the bottom, so $x_0 = x_3 = 0$. Assume that the only force acting on the masses, is gravity force. Find the stiffness matrix $K$ in the equation $Kx = f$, for the mass displacements. Solve for the displacements $x_1$ and $x_2$. 
2. If $b = (10, 5)$ and the matrix $A = [1 \ 2]^T$, what are the optimal vectors $Ax$ and $y$ such that $||Ax||^2 + ||y||^2 = ||b||^2$?
3. Find the best parabola $f(t) = a + b \, t + c \, t^2$ that approximates the given measurements $y = 2, 1, 3$ at $t = -1, 0, 1$.
Write the corresponding matrix $A$. Is $A^T A$ positive definite or semidefinite? Is it SPD matrix? Formulate and solve the least squares problem.
4. How far is it from the origin \((0, 0)\) to the plane \(y_1 + y_2 + y_3 = 6\)? Write down and minimize \(Q(y)\) subject to the given constraint.
5. Use Lagrange multipliers method to minimize $Q(y) = \frac{1}{2}(y_1^2 + \frac{1}{3}y_2^2)$ subject to $y_1 + y_2 = 5$. What is the dual quadratic and where is it maximized?