Math 5470 Nonlinear Dynamics and Chaos: Final Test

This test is closed book, closed notes test.

1. What is the dimension of the phase space for a system of two particles, which is described by the following equations:

$$\ddot{\mathbf{r}}_1 + 0.2\dot{\mathbf{r}}_1 = (\mathbf{r}_2 - \mathbf{r}_1)(\mathbf{r}_1 - \mathbf{r}_2)^2 t$$

$$\ddot{\mathbf{r}}_2 + 0.01\dot{\mathbf{r}}_2 = (\mathbf{r}_1 - \mathbf{r}_2)(\mathbf{r}_2 - \mathbf{r}_1)^2 t$$

Here $\mathbf{r}_1$ and $\mathbf{r}_2$ are two-dimensional vectors that characterize the location of the particles.

2. Consider the system $\dot{x} = \mu x + x^3 - x^5$ where $x(t)$ is a scalar function, μ is a real parameter. Plot the bifurcation diagram and find algebraic expressions for all the fixed points as μ varies. Calculate the value of μ at which the nonzero fixed points are born.

3. The system $\dot{x} = 1 - \mu x + x^2 y, \qquad \dot{y} = (\mu - 1)x - x^2 y$
is known to have a Hopf bifurcation at some critical value of the parameter μ.
What is this critical value?

4. Consider the system $\dot{x} = y + 2xy, \qquad \dot{y} = x + x^2 - y^2$.
Does it have a periodic orbit? Justify your answer.

5. Show that the fixed points of the Newton map $x_{n+1} = x_n - g(x_n)/g^{'}(x_n)$, where $g(x) = x^2 - 4$, are superstable.

6. Does the linearized system give a qualitatively correct picture of the phase portrait near the fixed point for the following system

$$\dot{x} = -x^3 - y, \quad \dot{y} = x$$

7. Find the similarity dimension of the subset of $[0,\ 1]$ consisting of real numbers that can be written without the digits 3 and 7 appearing anywhere in their decimal expansion.

8. Divide the closed interval $[0,\ 1]$ into four quarters. Delete the open second quarter from the left. This produces S_1. On the next step, divide each of the quarter intervals into four quarters, and in each of the subintervals, delete the open second quarter from the right.
Repeat the construction: i.e., generate S_{n+1} from S_n by deleting the second left or second right quarter of each of the intervals in S_n.
a. Sketch the sets S_1, S_2, S_3.
b. Compute the box dimension of the limiting set S_∞.
c. Is the set S_∞ self-similar?