

MATH 1030: Homework 6

due March 1, 2012

Instructions: Do the following problems on a separate sheet of paper. Show all of your work.

Fereidoun M. Esfandiary, a futurist and transhumanist, was born in Iran in 1930. He lived in 17 countries by the time he was 11. In 1970, he changed his name to FM-2030 because “conventional names define a person’s past: ancestry, ethnicity, nationality, religion. [...] The name 2030 reflects my conviction that the years around 2030 will be a magical time.”

Problem 1

With the sales from FM-2030’s first nonfiction book *Upwingers: A Futurist Manifesto* in 1973, he decided to open a savings account. Being a transhumanist, FM wanted to get the most for his money. His bank offered an account with an APR of 7.43% compounded annually or an account with an APR of 7.37% compounded quarterly.

- (a) Compute the APY for both of the accounts to determine which account is the better investment.

The first account is compounded annually, so $APY=APR=7.43\%$. For the second account the APY is

$$APY = \left(1 + \frac{APR}{n}\right)^n - 1 = \left(1 + \frac{.0737}{4}\right)^4 - 1 = .0757 = 7.57\%$$

so the account that is compounded quarterly is better.

- (b) If FM-2030 deposited \$15,000 into the better account, how much did he have in the account 27 years later in the year 2000?

If he deposited \$15,000 into the account where the interest is compounded quarterly at an APR of 7.37%, then 27 years later he would have

$$A = P \left(1 + \frac{APR}{n}\right)^{n \cdot Y} = 15,000 \left(1 + \frac{.0737}{4}\right)^{4 \cdot 27} = \$107,753$$

- (c) Of the total amount in the year 2000, what percentage was deposited and what percentage was earned in interest?

He deposited a total of only \$15,000 and ended up with \$107,753. So $\frac{15,000}{107,753} = .139 = 13.9\%$ was deposited and the rest, 18.5% was earned in interest.

Problem 2

As a futurist, FM-2030 decided that cryopreservation was for him. The Alcor Life Extension Foundation in Scottsdale, Arizona charges \$150,000 to cryogenically freeze an entire body.

- (a) If he wanted to have the \$150,000 by the year 1995 and his savings account compounded continuously at an apr of 5.8%, how much did he need to deposit in 1975 in order to reach his goal?

We must find what initial investment, P , would yield \$150,000 if continuously compounded for 20 years at an APY of 5.8%. This leads to the equation

$$\$150,000 = Pe^{.058 \cdot 20} \quad \implies \quad P = \frac{\$150,000}{e^{.058 \cdot 20}} = \$47,022$$

- (b) If he wanted to have the \$150,000 by the year 2000 and his savings account compounded monthly at an APR of 4.8%, how much did he need to deposit in 1975 in order to reach his goal?

FM-2030 wants to make a one-time deposit in 1975 so that he has \$150,000 twenty-five years later in the year 2000. Thus,

$$\$150,000 = P \left(1 + \frac{.048}{12}\right)^{12 \cdot 25} \quad \implies \quad P = \frac{\$150,000}{\left(1 + \frac{.048}{12}\right)^{12 \cdot 25}} = \$45,287$$

Problem 3

In addition to the \$150,000 freezing fee, there is also a yearly membership fee of \$500. FM-2030 knew that he might be in cryopreservation for hundreds of years, so he needed a savings account that earns \$500 in interest every year.

- (a) If his bank account has an APR of 5.5%, how much money does he need in the account in order to generate the \$500 in interest every year.

The account earns 5.5% in interest every year. So the question is “\$500 is 5.5% of what number?”

$$\frac{500}{P} = .055 \quad \implies \quad P = \$9090$$

So an account balance of \$9090 would generate \$500 in interest every year.

- (b) Suppose that his account with the APR of 5.5% compounds daily. If he decided to start the account in 1980, how much should he have deposited in order to have the amount calculated in part (a) by the year 2000?

FM-2030 wants to make a one time deposit in 1980 so that the account has \$9090 twenty years later in the year 2000. Using the interest formula, we have

$$\$9090 = P \left(1 + \frac{.055}{365}\right)^{20 \cdot 365} \quad \implies \quad P = \$3026$$

So FM-2030 should have deposited \$3026 in 1980 to have the required amount in 2000.

Problem 4

Unfortunately, FM-2030 did not make it to year 2030 alive. He died of pancreatic cancer on July 8, 2000. However, he was cryogenically frozen as planned. Suppose he took his remaining \$100,000 and deposited it in a bank account that compounds every 2 months at an APR of 6%.

- (a) If FM-2030 is unfrozen in the year 2415, how much will he have in his bank account?

We use the compound interest formula to compute

$$A = \$100,000 \left(1 + \frac{.06}{6}\right)^{6 \cdot 415} = \$5.757 \times 10^{15}$$

So FM-2030 has 5.757 quadrillion dollars.

- (b) What percent more than the deposited \$100,000 is the account total in the year 2415?

The relative change is

$$\frac{\$5.757 \times 10^{15} - \$100,000}{\$100,000} = 5.757 \times 10^{10}\%$$

§4B Exercise 46

The account is earning *simple* interest on \$1,800 at an APR of 3.2%. So each year, the account earns $\$1,800 \times .032 = \57.60 . Over five years the account will earn $\$57.60 \times 5 = \288 . So the account balance will be $\$1,800 + \$288 = \$2,088$.

§4B Exercise 48

We have the following table

Year	Ariel's Annual Interest	Ariel's Balance	Travis's Annual Interest	Travis's Balance
1	\$150	\$5150	\$150	\$5150
2	\$150	\$5300	\$154.50	\$5304.50
3	\$150	\$5450	\$159.14	\$5463.64
4	\$150	\$5600	\$163.91	\$5627.55
5	\$150	\$5750	\$168.83	\$5796.37

§4B Exercise 52

Applying the formula, we have $A = \$3,000(1 + .04)^{12} = \$4,803.10$.

§4B Exercise 56

Applying the compound interest formula, we have $A = \$2,000 \left(1 + \frac{.03}{365}\right)^{365 \cdot 5} = \$2,323.65$.

§4B Exercise 74

We want to have \$25,000 eight years from now, so

$$\$25,000 = P \left(1 + \frac{.045}{4}\right)^{4 \cdot 8} \implies P = \$17,477$$

§4B Exercise 82

Ten years from now, José will have

$$A = \$1,500(1 + .056)^{10} = \$2,586.61$$

While Martha will have

$$A = \$1,500(1 + .057)^{10} = \$2,611.21$$

A similar calculation shows that after 30 years, José will have \$7,691.46 while Martha will have \$7,912.99.

§4B Exercise 89

With the first account we would need to deposit

$$\$120,000 = P(1 + .05)^{30} \quad \implies \quad P = \$27,765.29$$

And with the second account, we would have to deposit

$$\$120,000 = Pe^{.048 \cdot 30} \quad \implies \quad P = \$28,431.33$$