

MATH 1030: Homework 4

due February 3, 2014

Instructions: Do the following problems on a separate sheet of paper. Show all of your work.

§2A Exercise 39

We know that $1 \text{ ft} = 12 \text{ in}$. How about 1 square foot? What is the definition of a square foot? It is the area of a square whose sides have length equal to 1 foot. Similarly, a square inch is the area of a square whose sides have lengths of 1 inch. If we have a square foot, each of its sides are 12 inches long. So how many little squares with size 1 in^2 can we fit in the square? We can cut it up into a grid that has size 12×12 . Then each of the squares in the grid will be the size of a square inch and there will be $12 \times 12 = 144$ squares. So $1 \text{ ft}^2 = 144 \text{ in}^2$. Another way to see this is by doing a unit conversion from ft^2 to in^2

$$1 \text{ ft}^2 = 1 \text{ ft} \cdot \text{ft} \left(\frac{12 \text{ in}}{1 \text{ ft}} \right) \left(\frac{12 \text{ in}}{1 \text{ ft}} \right) = 12 \cdot 12 \text{ in} \cdot \text{in} = 144 \text{ in}^2$$

We can write this conversion factor in three ways as

$$1 \text{ ft}^2 = 144 \text{ in}^2 \quad \text{or} \quad \frac{144 \text{ in}^2}{1 \text{ ft}^2} = 1 \quad \text{or} \quad \frac{1 \text{ ft}^2}{144 \text{ in}^2} = 1$$

§2A Exercise 42

The area in square yards is $12 \cdot 20 = 240 \text{ yd}^2$. Now we convert this to square feet

$$240 \text{ yd}^2 = 240 \text{ yd} \cdot \text{yd} \left(\frac{3 \text{ ft}}{1 \text{ yd}} \right) \left(\frac{3 \text{ ft}}{1 \text{ yd}} \right) = 240 \cdot 3 \cdot 3 \text{ ft} \cdot \text{ft} = 2160 \text{ ft}^2$$

§2A Exercise 55

If the airliner travels 45 miles in 5 minutes, its speed is $\frac{45 \text{ mi}}{5 \text{ min}} = 9 \text{ mi/min}$. We want to convert this to miles per hour. So

$$\frac{45 \text{ mi}}{5 \text{ min}} \left(\frac{60 \text{ min}}{1 \text{ hr}} \right) = 540 \text{ mph}$$

§2A Exercise 79

(a) Kevin Garnett played 80 games and earned a total of \$24,750,000. In each game he made

$$\frac{\$24,750,000}{80 \text{ games}} = \$309,375$$

- (b) If he played all 48 minutes of every game, then he earned

$$\frac{\$309,375}{48 \text{ min}} = \$6,445 \text{ perminute}$$

- (c) If he played $\frac{2}{3}$ of every game, then he played $48 \cdot \frac{2}{3} = 32$ minutes of every game. So he earned

$$\frac{\$309,375}{32 \text{ min}} = \$9,668 \text{ perminute}$$

- (d) If he practiced or trained for 40 hours for every game, then he would have worked for a total of $(40 \text{ hrs/game}) \cdot (80 \text{ games}) = 3200 \text{ hrs}$. So his salary would be

$$\frac{\$24,750,000}{3200 \text{ hrs}} = \$7,734 \text{ perhour}$$

Note that there is some ambiguity in this question. We could also interpret it as asking us to include his playing time, in which case he would have “worked” a total of 40 hours and 48 (or 32) minutes per game.

§2A Exercise 81

- (a) We have to convert 1.4 GB into pages of text.

$$1.4 \text{ GB} = 1,400,000,000 \text{ bytes} \left(\frac{1 \text{ char}}{1 \text{ byte}} \right) \left(\frac{1 \text{ page}}{2,000 \text{ char}} \right) = 700,000 \text{ pages}$$

- (b) The Kindle can hold

$$700,000 \text{ pages} \left(\frac{1 \text{ book}}{500 \text{ pages}} \right) = 1,400 \text{ books}$$

Problem 1: Pen Pals

Help pen pals Geraldine and Andryushka (both age 11) understand each other’s country by doing the following problems. Show all your work.

- (a) Andryushka has been studying the oil crisis and writes to Geraldine to tell her that the average price of a gallon of gasoline in Moscow is 23.49 Russian rubles (RUR) per liter. Help Geraldine figure out how many dollars per gallon it would be.
(1 dollar = 29.93 RUR and 1 gallon = 3.785 liters)

We are interested in the price of gasoline per unit volume. So we write it down in the units we have, then convert.

$$\frac{23.49 \text{ RUR}}{1 \text{ litre}} \left(\frac{1 \text{ USD}}{29.93 \text{ RUR}} \right) \left(\frac{3.785 \text{ litres}}{1 \text{ gal}} \right) = \frac{2.97 \text{ USD}}{1 \text{ gal}}$$

- (b) Andryushka has been learning how to quilt recently and asks Geraldine (an avid quilter) if she has a favorite pattern. The pattern Geraldine sends him asks for 100 square feet of material. Help Andryushka know how many square meters of fabric he needs to have.
(1 meter = 3.28 feet)

We have to convert 100 square feet into square meters. We have to be careful to cancel out both factors of feet in the numerator.

$$100 \text{ ft}^2 \left(\frac{1 \text{ m}}{3.28 \text{ ft}} \right) \left(\frac{1 \text{ m}}{3.28 \text{ ft}} \right) = 9.29 \text{ m}^2$$

- (c) Geraldine has recently learned that there are 7.48 gallons in 1 cubic foot. She excitedly writes to Andryushka and asks him how many liters are in a cubic meter. Help Andryushka perform this calculation.

Liters and cubic meters are both measurements of volumes. We have to convert 1 m^3 into liters.

$$1 \text{ m}^3 \left(\frac{3.28 \text{ ft}}{1 \text{ m}} \right)^3 \left(\frac{7.48 \text{ gal}}{1 \text{ ft}^3} \right) \left(\frac{3.7851}{1 \text{ gal}} \right) = 999.051$$

Note that we have to multiply by $\frac{3.28 \text{ ft}}{1 \text{ m}}$ three successive times to cancel out each factor in the numerator.

- (d) “It sure is cold here in Russia,” Andryushka writes. “This last winter, the average temperature where I lived was negative 15 degrees Celcius (-15°).” Help Geraldine convert the temperature into Fahrenheit.

Recall the formula ($F = 1.8C + 32$), we convert -15° C to Fahrenheit

$$F = 1.8(-15) + 32 = 5^\circ \text{ F}$$

- (e) “Is there a temperature where the number of degrees Celcius is equal to the number of degrees Fahrenheit?” Geraldine inquires. “In fact, there is only one such temperature,” Andryushka replies. Help Geraldine figure out what that temperature is.

We want to find a temperature that is the same in both Fahrenheit and Celcius. If you’re unsure of what is happening here, consider this: The freezing point of water is 32° F and 0° C , so at this temperature, the “Fahrenheit scale” reads a larger number than the “Celcius scale.” What about at absolute zero? At absolute zero, the temperature is -273.15° C or -459.7° F . At this temperature, the situation is the opposite. The “Celcius scale” reads a larger number than the “Fahrenheit scale”. Then there must be some temperature in between where both “scales” read the same value. Suppose the temperature where both scales read the same is x degrees Celcius. Then if we convert to Fahrenheit using the formula, we will get the same temperature back again. So we have

$$x = 1.8x + 32 \quad \implies \quad x - 1.8x = 32 \quad \implies \quad -0.8x = 32 \quad \implies \quad x = -40$$

- (f) The average price of gas in the United States is \$3.67 per gallon. Suppose that Andryushka's family is visiting and that they rent a car that gets 13 kilometers per liter. If they have to travel a total of 250 miles, how much will they spend on gasoline in RUR?
(1 dollar = 29.93 RUR, 1 mile = 1.609 kilometers, 1 gallon = 3.785 liters)

We can convert 13 kilometers per liter into miles per gallon. We have

$$13 \frac{\text{km}}{\text{liter}} \left(\frac{3.785 \text{ liters}}{1 \text{ gal}} \right) \left(\frac{1 \text{ mi}}{1.60 \text{ km}} \right) = 30.75 \frac{\text{miles}}{\text{gal}}$$

Now if we travel 250 miles using 30.75 miles per gallon, how many gallons of gasoline do we use. There is only one way to combine these two numbers to get something that has units of gallons. We have

$$250 \text{ mi} \left(\frac{1 \text{ gal}}{30.75 \text{ mi}} \right) = 8.13 \text{ gal}$$

At a price of \$3.67 per gallon, they will spend $\frac{\$3.67}{1 \text{ gal}} \cdot 8.13 \text{ gal} = \29.84 . We can now convert to RUR.

$$\$29.84 \left(\frac{29.93 \text{ RUR}}{\$1} \right) = 893 \text{ RUR}$$

There are several alternative ways to approach this problem. If you came up with a different way, that is great.