Math 3220 — Study guide for final exam

1. This study guide only covers the chapter on **integration**, but the final exam is comprehensive. To review other chapters, do all problems in the study guides for Midterm 1 and Midterm 2.

2. Let R be the rectangle $[0,1] \times [0,1] \subset \mathbb{R}^2$, and let P be the partition of R obtained by partitioning each factor of $[0,1] \times [0,1]$ via $\{0, \frac{1}{4}, \frac{2}{4}, \frac{3}{4}, 1\}$. If $f(x,y) = x^2 y$, compute $U(f,P)$ and $L(f,P)$.

3. Show that $f(x,y) = x^2 y$ is integrable on $[0,1] \times [0,1]$. (Use similar partitions as in the previous problem, but this time coming from partitions of $[0,1]$ of the form $\{0, \frac{1}{n}, \frac{2}{n}, \ldots, 1\}$).

4. The following statements are all false. Give counterexamples.

   (a) If a set is countable, it is of volume zero.

   (b) If $\{f_n\}$ is a sequence of continuous real valued functions on $[0,1]$ converging pointwise to a function f, then

$$\lim_{n\to\infty} \int_0^1 f_n(x)\,dx = \int_0^1 f(x)\,dx.$$

   (c) If f is an integrable function on a Jordan region A, then $\left|\int_A f(x)\,dV(x)\right| = \int_A |f(x)|\,dV(x)$.

5. Suppose A and B are sets of volume zero in $\mathbb{R}^d$. Show that $A \cup B$ has volume zero.

6. Suppose that f and g are integrable on a Jordan region $A \subset \mathbb{R}^d$, and that $f(x) \le g(x)$ for all $x \in A$. Prove that $\int_A f(x)\,dV(x) \le \int_A g(x)\,dV(x)$.

7. Suppose that f and g are integrable on a rectangle region $R \subset \mathbb{R}^d$. Show that $f+g$ is also integrable on R, and that

$$\int_R (f+g)(x)\,dV(x) = \int_R f(x)\,dV(x) + \int_R g(x)\,dV(x).$$

Extra homework problems (not to turn in):

Section 10.3: 9, 10, 13

Section 10.4: 2, 7