Sample Midterm 3

1

Solve the wave equation on a disk of radius 2 with $c = 1$ and initial conditions:

$$u(r,\theta,0) = 3\ J_0\left(\tfrac{\alpha_{0,2}}{2} r\right) - 2\ J_0\left(\tfrac{\alpha_{0,1}}{2} r\right)$$

$$\frac{\partial u}{\partial t}(r,\theta,0) = J_0\left(\tfrac{\alpha_{0,3}}{2} r\right) - 4\ J_0\left(\tfrac{\alpha_{0,1}}{2} r\right)$$

2

Solve the wave equation on a disk of radius 1 with $c = 1$ and initial conditions:

$$u(r,\theta,0) = J_1(\alpha_{1,2}\, r)\ \sin\theta - 2\, J_0(\alpha_{0,1}\, r) + 3\, J_2(\alpha_{2,2}\, r)\ \cos 2\theta$$

$$\frac{\partial u}{\partial t}(r,\theta,0) = 0$$

3

Solve the wave equation on a disk of radius 1 with $c = 1$ and initial conditions:

$$u(r,\theta,0) = 0$$

$$\frac{\partial u}{\partial t}(r,\theta,0) = r\sin\theta - r^2\cos 2\theta$$

4

Find a solution to Dirichlet's problem on a disk of radius 2 and boundary condition:

$$u(2,\theta) = \begin{cases} \pi - \theta & \text{if } 0 < \theta < \pi \\ 0 & \text{if } \pi < \theta < 2\pi \end{cases}$$

5

Find a solution to Dirichlet's problem on a disk of radius 1 and boundary condition:

$$u(1,\theta) = \sin 2\theta + \cos 2\theta$$

Determine which points in the disk have temperature 0 (i.e., determine which points belong to the isotherm given by $T = 0$).

6

Using the definition for the Bessel function of order n,

$$J_n(x) = \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+n)!}\left(\frac{x}{2}\right)^{2k+n},$$

show that:

$$\frac{d}{dx}\left[J_0(x)\right] = -J_1(x)$$

Use this fact to show that the local maxima and minima of $J_0(x)$ occur at the zeros of $J_1(x)$.