Sample Midterm 2

1

Consider the following PDE:

$$\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^3 u}{\partial x^3}$$

Use the method of separation of variables to transform the previous PDE into a pair of ODEs.

2

A bar of length 10 with insulated lateral ends and no internal sources of heat has its $x = 0$ end held at a constant 0 degrees and its $x = 1$ end held at a constant 50 degrees. The initial temperature of the bar is a constant 100 degrees. The constant c in the heat equation is 4 for this bar.

a) Find the steady-state temperature distribution.
b) Find the function $u(x, t)$ that describes the temperature of point x of the bar at time t.

3

Solve the heat equation in a 1×1 square plate with zero boundary conditions and the following initial condition:

$$u(x, y, 0) = \begin{cases} 100 & \text{if } 0 < x \leq \frac{1}{2} \\ 0 & \text{if } \frac{1}{2} < x < 1 \end{cases}$$

Assume that the constant c in the heat equation is 1.

4

Determine the steady-state temperature distribution in a 1×1 square plate with boundary conditions

$$f_1(x) = 0 \qquad f_2(x) = 10 \qquad g_1(y) = g_2(y) = 10y$$

5

Determine the steady-state temperature distribution in a 2×2 square plate with boundary conditions

$$f_1(x) = f_2(x) = g_2(y) = 0 \qquad g_1(y) = \sin \tfrac{\pi}{2} y + 3 \sin 2\pi y$$

6

Find a solution to the one-dimensional wave equation with $c = 1$ and $L = \pi$,

$$\frac{\partial^2 u}{\partial^2 t} = \frac{\partial^2 u}{\partial x^2} \qquad \text{for} \quad 0 \leq x \leq \pi, \quad t \geq 0,$$

satisfying the usual boundary conditions,

$$u(0, t) = 0, \qquad u(\pi, t) = 0, \qquad \text{for } t \geq 0,$$

and satisfying the following initial conditions:

$$u(x, 0) = x, \qquad \frac{\partial u}{\partial t}(x, 0) = 1 - x^2, \qquad \text{for } 0 \leq x \leq \pi.$$

Note: the actual midterm will probably be shorter. You are allowed to bring a crib sheet to the midterm (one page, both sides).