NAME:

Sample Midterm 1

Partial Differential Equations for Engineering Students

Math 3150, Spring 2011

Problem 1 (20 pts)	
Problem 2 (20 pts)	
Problem 3 (20 pts)	
Problem 4 (20 pts)	
Problem 5 (20 pts)	
Total (100 pts)	

1

Verify that the function

$$u(x,t) = \frac{1}{\sqrt{t}} e^{-\frac{x^2}{4t}}$$

is a solution of the heat equation

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \qquad -\infty < x < \infty, \quad t > 0$$

2

Recall that the general solution to the partial differential equation

$$\frac{\partial u}{\partial t} + \frac{\partial u}{\partial x} = 0$$

is of the form

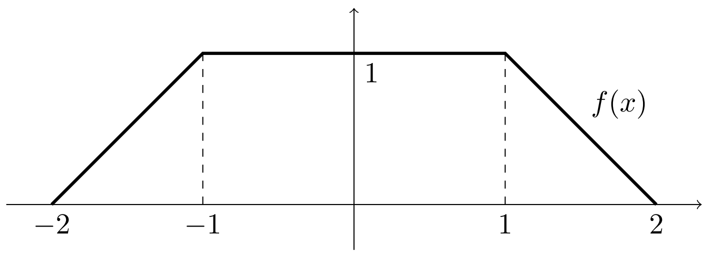
$$u(x,t) = f(x-t),$$

where f is an arbitrary differentiable function of a single variable. Find the particular solution to the above differential equation corresponding to the following initial condition:

$$u(x,0) = \frac{\cos x}{e^x}.$$

3

Let $f(x)$ be the periodic function with period 4 whose graph in the interval $(-2, 2)$ is the following:



Find the Fourier series of $f(x)$.

4

Find the cosine series expansion of $f(x) = e^{-x}$ in the interval $0 \le x \le \pi$.

5

Find a solution to the one-dimensional wave equation with $c = 1$ and $L = \pi$,

$$\frac{\partial^2 u}{\partial^2 t} = \frac{\partial^2 u}{\partial x^2} \qquad \text{for} \quad 0 \le x \le \pi, \quad t \ge 0,$$

satisfying the usual boundary conditions,

$$u(0,t) = 0, \qquad u(\pi,t) = 0, \qquad \text{for } t \ge 0,$$

and satisfying the following initial conditions:

$$u(x,0) = \cos x, \qquad \frac{\partial u}{\partial t}(x,0) = 0, \qquad \text{for } 0 \le x \le \pi.$$

Useful Integrals

$$\int x\cos ax\;dx = \frac{1}{a^2}\cos ax + \frac{x}{a}\sin ax + C$$

$$\int x\sin ax\;dx = \frac{1}{a^2}\sin ax - \frac{x}{a}\cos ax + C$$

$$\int x^2\cos ax\;dx = \frac{2x\cos ax}{a^2} + \frac{a^2x^2-2}{a^3}\sin ax + C$$

$$\int x^2\sin ax\;dx = \frac{2x\sin ax}{a^2} - \frac{a^2x^2-2}{a^3}\cos ax + C$$

$$\int \cos ax\;\sin bx\;dx = \frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)} + C \qquad (a^2 \neq b^2)$$

$$\int e^{ax}\;\cos bx\;dx = \frac{e^{ax}}{a^2+b^2}\,(a\cos bx + b\sin bx) + C \qquad (a^2+b^2 \neq 0)$$

$$\int e^{ax}\;\sin bx\;dx = \frac{e^{ax}}{a^2+b^2}\,(a\sin bx - b\cos bx) + C \qquad (a^2+b^2 \neq 0)$$