Sample for Midterm 1

Math 1220, Calculus II

1

Compute the derivative of the following functions:

$$f(x) = \frac{\ln x^2}{\ln x^3}, \qquad g(x) = \ln(\arccos x + \arcsin x), \qquad h(x) = (\sinh 4x)\,(\coth 2x).$$

2

Compute the following integrals:

(a) $\displaystyle\int_e^{e^e} \frac{1}{x \ln x}\,dx$ (b) $\displaystyle\int_0^{\pi/2} \frac{\cos x}{1+\sin^2 x}\,dx$ (c) $\displaystyle\int (\cot x)\,\ln(\sin x)\,dx$

3

Rewrite the following expressions without using trigonometric or inverse trigonometric functions:

(a) $\arcsin\left(\sin\left(-\frac{8\pi}{7}\right)\right)$ (b) $\sin\left(\arccos\left(\frac{1+\sqrt{3}}{2\sqrt{2}}\right)\right)$ (c) $\csc(\arctan(x))$

4

Compute $(f^{-1})'(0)$ for the following functions:

(a) $f(x) = \displaystyle\int_1^x \frac{\sin t}{t}\,dt$ (b) $f(x) = \displaystyle\int_0^{\ln x} t^2 + 1\;dt$

5

The half-life of a radioactive substance is 15 years. How long does it take for the substance to decay to $\frac{1}{8}$ of its original size?

6

Find the half-life of a radioactive substance if its original size is 100 grams and decays to 98 grams in 2 months.

7

A tank initially contains 100 gallons of brine with 1% salt. A pipe pumps a brine solution with 0.5% salt at 2 gallons per minute. The solution gets mixed in the tank and pumps out at a rate of 1 gallons per minute. What is the percentage of salt in the tank after 100 minutes?

8

Solve the following differential equations:

(a) $y' = \dfrac{3y+1}{1+x^2}, \quad y(0) = 0$

(b) $y' - \dfrac{2y}{x} - 3x^4 = 0, \quad y(1) = 3$

(c) $y'' - 7y' + 6 = 0,\ y(0) = 10,\ y'(0) = 20$

(d) $y'' - 2y + 2 = 0,\ y(0) = 1,\ y\left(\dfrac{\pi}{2}\right) = 0$