Final Review

Math 1220, Calculus II

1

Evaluate the following integrals:

$$\int \frac{dx}{\sqrt{x}(1+\sqrt{x})^2} \qquad \int_0^2 \frac{dx}{(x-1)^2} \qquad \int_0^4 \frac{x\,dx}{\sqrt{x^2+9}} \qquad \int x\ \sin x\ dx$$

2

Solve the differential equation $\frac{dy}{dt} + 4y = 8$ given the initial condition $y = 4$ when $t = 0$.

3

What is the sum of $1 + \frac{\pi}{4} + \frac{\pi^2}{16} + \frac{\pi^3}{64} + \ldots$?

4

Find the first four non-zero terms of the Maclaurin series of $f(x) = e^{x^4}$, and use them to estimate the following integral:

$$\int_0^{\frac{1}{2}} e^{x^4}\ dx.$$

5

Evaluate the following integrals:

$$\int \frac{x^2}{x^3+1}\ dx \qquad \int y\ e^{-2y}\ dy \qquad \int_0^{\pi} (\sin\theta + \cos\theta)\ d\theta \qquad \int x^2\ e^{x^3}\ dx \qquad \int \frac{x+1}{x(x-1)(x^2+1)^2}\ dx$$

6

For a new radioactive substance, it is found that after 10 years, 5% of the substance has decayed. Find the half life of the substance.

7

Find the convergence set for the series

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{n!}$$

8

Determine whether the following series converges absolutely, conditionally, or diverges.

$$\sum_{n=1}^{\infty} \frac{(-1)^n\ n^2\ 3^{n+1}}{e^{2n}}$$

9

Find a power series for $F(x)$, and state its radius of convergence.

$$F(x) = \int x\, e^{x^3}\, dx$$

10

Find $\frac{dy}{dx}$ for each function. Do not bother simplifying your answers, and make sure your answers are given as $y =$ some function of x.

$$y = \frac{\ln(3x+1)}{\cos\sqrt{x}} \qquad\qquad y = (x^2+2)^{1+x}$$

11

Solve the following differential equation:

$$\frac{dy}{dx} = \frac{x^3 - 4y}{x}$$

12

Evaluate the following integrals.

$$\int \frac{\sqrt{4x^2+9}}{x^4}\, dx \qquad \int_2^4 \frac{dx}{\sqrt{x-2}} \qquad \int_0^{\frac{\pi}{3}} x\, \cos 3x\, dx \qquad \int \frac{2x+3}{x^3+x}\, dx$$

$$\int \cos^3 x\, dx \qquad \int_{-\infty}^{0} 8\, x^2\, e^{-x^3}\, dx \qquad \int_0^1 \frac{4y}{\sqrt{y^2+6}}\, dy$$

13

Find the following limits, if they exist.

$$\lim_{x\to 0} \frac{\sin 5x}{\tan 4x} \qquad\qquad \lim_{x\to\infty} (2x)^{\frac{1}{3x}}$$

14

Consider the sequence given by

$$a_n = \frac{5n}{\sqrt{5n^2-3}}.$$

Find the first three terms of the sequence, determine if it converges or diverges, and, if it converges, find its limit.

15

Determine if each series is absolutely convergent, conditionally convergent or divergent. (Show your work and state which tests you are using to get to your answer.)

$$\sum_{n=1}^{\infty} \frac{\sqrt{3n}}{n^3+5} \qquad\qquad \sum_{n=1}^{\infty} \frac{n-2}{4n+1} \qquad\qquad \sum_{n=1}^{\infty} \frac{n^4\, (-3)^n}{(n+2)!}$$

16

Find a power series that represents the function

$$f(x) = \frac{1}{1-2x} + e^{-3x},$$

and state its radius of convergence.

17

Consider the function

$$f(x) = \frac{1}{3+x}.$$

Find its Taylor polynomial of order 3 based at $a = 2$. Approximate $f(2.3)$ using this Taylor polynomial, and find a bound for the error in your approximation.

18

Determine the type of conic that is represented by each of the following equations.

$$y^2 - 5x - 4y - 6 = 0 \qquad 4x^2 - 4y^2 + 8x + 12y - 5 = 0 \qquad 9x^2 + 4y^2 + 72x - 16y + 124 = 0$$

19

Find the area of the region enclosed by $r = 1 + \sin\theta$.

20

Find the area of the region outside $r = 2 + 2\cos\theta$ and inside $r = 2$

21

For Cartesian coordinates $(-3\sqrt{3}, 3)$, find three different ways to represent this point in polar coordinates.

22

Find the slope of the tangent line to the curve $r = 2 + 3\sin\theta$ at the point where $\theta = \pi/4$.

23

Find the equation in polar coordinates for the curve

$$x^2 + (y-2)^2 = 2$$

Using the polar equation, find the area of the region bounded by the curve. (Hint: You may want to graph the function to ensure you get the correct integration bounds.)
