5. You may assume that there exist constants C_1 and C_2 such that the following inequalities hold for all $u \in H^1(U)$:

$$\left(\int_{\partial U} u^2 \, dS \right)^{1/2} \leq C_1 \|u\|_{H^1(U)},$$

and

$$\|u\|_{L^2(U)}^2 \leq C_2 \left(\int_{\partial U} u^2 \, dS + \|Du\|_{L^2(U)}^2 \right).$$

15. Hints:

   a. Note $\lambda(\tau) = \displaystyle\int_{U(\tau)} |Dw(\tau)|^2 \, dx$.

   b. Use Theorem from C.4.

   c. Integrate by parts and interchange derivatives.

   d. Use the eigenvalue equation to replace $-\triangle w$.

   e. Notice that $\displaystyle\int_U w\dot{w} \, dx$ simplifies considerably since $\displaystyle\int_{U(\tau)} w^2(\tau) \, dx \equiv 1$.

   f. Notice that $w = 0$ on ∂U, so the gradient of w is zero in directions tangent to ∂U.