

Project 1: Modeling the Dynamics of a Freshwater Aquarium

Math 5740, Spring 2003

In the process of trying to decrease losses of fish due to improper water conditions, a large tropical fish retailer has undertaken a project to mathematically model the dynamics of its freshwater display aquariums.

Each aquarium holds 50 gallons of water. The water is carefully maintained at 82 degrees F at all times. A pump constantly circulates the water through a carbon/polyester filter element at a rate of 100 gallons/hour. As one result of the pumping, the water is *well-mixed*: every measurable property of the water is essentially the same at all spatial locations within the aquarium.

Fish are maintained in each aquarium at some volume v . In general, v varies with time as fish grow, are sold, and are replaced with new fish. Since tank space is expensive, the store is interested in keeping fish volume high.

A certain amount a of food is added (all at once) to each tank at the same time every day. The food is carefully metered; however, due to differences among fish species and other factors, there is no known exact “optimal” amount of food for a given volume v of fish. It is known that *too much* food can lead to serious chemical imbalances in the tank, ultimately resulting in disease or death for the fish.

Every two weeks, the tanks are cleaned. This involves draining 30% of the water in each tank and replacing it with “fresh” dechlorinated tap water. Also, accumulated sediment in the gravel bed is (mostly) removed, and algae buildup is scrubbed away. Every sixteen weeks, the filters are removed and all filter material is replaced.

Tropical fish are very susceptible to disease, fungi, and parasites. Their resistance to these problems is greatly decreased by physiological stress induced by improper water conditions, such as excessive concentrations of ammonia or nitrites. Other important factors are pH and oxygen levels.

Aquarium chemistry is very complicated, involving many coupled chemical and biological processes. Perhaps the most important part of overall aquarium chemistry is the *nitrogen cycle*. In this process, ammonia (NH_3) from fish waste products is converted by *nitrosomonas* bacteria into nitrite (NO_2^-), also harmful to fish. The nitrites are then converted by *nitrobacter* bacteria into relatively harmless nitrate (NO_3^-). If the appropriate bacteria are not present (usually they become established in filter beds) in sufficient

numbers, then ammonia and/or nitrite concentrations will quickly reach unacceptable levels and the fish will become weakened or die.

Our job is to produce a model which predicts ammonia, nitrite, and nitrate concentrations as a function of time, for given fish volume, food amount, and cleaning schedule. Note that in order to obtain the three chemical concentrations listed, it may (or may not) be important to include many other quantities or parameters in your model. Verify your model against the qualitative and quantitative data contained in the references. Finally, explore the dynamics of your model as fish volume, feeding amounts and times, and cleaning times are varied. Use your results to formulate recommendations for management on safe feeding and cleaning schedules, control of fish volume, etc. Keep in mind that management wants to keep fish volume high in each tank and reduce costs by cleaning infrequently. Also identify any potentially dangerous scenarios indicated by the dynamics of your model.