

Math 3210 Exam I Sample Questions

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Caveat: This is only intended to give you some idea of the types of questions to expect on Exam I. Other kinds of problems related to the course material are still fair game.

1. Suppose P and Q are mathematical statements. Construct a truth table showing the truth values of the following statements:
 - (a) $(P \text{ or } Q) \Rightarrow (P \text{ and } Q)$.
 - (b) $P \text{ and } [\sim (Q \text{ or } P)]$.
2. Negate the following statements (note these are not necessarily true):
 - (a) For every $x \in \mathbb{R}$, there exists a rational number q such that $q + x$ or qx is irrational.
 - (b) $(\epsilon > 0 \text{ and } x > 14)$ implies $\epsilon x > 0$.
3. Using induction, prove that the sequence defined by $a_1 = 1$ and $a_{n+1} = \frac{a_n}{3}$ for $n \in \mathbb{N}$, satisfies $a_n = \frac{1}{3^{n-1}}$ for all $n \in \mathbb{N}$.
4. Find the complement in \mathbb{R} of the set $\bigcup_{s \in [1,2)} [-s, s]$, and prove that your answer is correct.
5. Assume $f : A \rightarrow \mathbb{R}$ and $g : B \rightarrow \mathbb{R}$ are functions, where A and B are nonempty sets of real numbers, and $A \subset B$. If $f(x) \leq g(x)$ for all $x \in A$, prove that $\sup_A f \leq \sup_B g$.
6. Let $\{a_n\}$ be a sequence of real numbers. Using the definition of the limit, prove: $(a_n \rightarrow a \text{ implies } (-1)^n a_n \rightarrow a)$ if and only if $a = 0$.