

## Math 3210 Exam I

October 2, 2006

*Instructions:* Write clear, concise proofs (on problems requiring proofs) for full credit. *You may use the results of theorems, examples, and problems we proved in class or on homework.*

**1.** (10 points). Let  $P$  and  $Q$  be mathematical statements. Construct a truth table showing the truth values of:  $[\sim (Q \text{ or } P)]$  implies  $P$ .

**2.** (10 points). Negate the statement: If  $(x > 0$  and  $y = 1)$  then there exists  $N \in \mathbb{N}$  such that  $Nxy = 5$ .

3. (20 points). Assume  $r$  is a real number with  $0 \leq r < 1$ . Using induction, prove that  $\sum_{j=1}^n r^j = \frac{r - r^{n+1}}{1 - r}$  for all  $n \in \mathbb{N}$ .

4. (20 points). Prove that for every real number  $x$ , there exists a sequence  $\{q_n\}$  of rational numbers with  $q_n < x$  for all  $n \in \mathbb{N}$ , and  $q_n \rightarrow x$ . You may use without proof the fact (from homework) that there is a rational number between every distinct pair of real numbers.

**5.** (20 points). Let  $\{a_n\}$  and  $\{b_n\}$  be sequences of real numbers, with  $a_n \rightarrow a$ . Construct a new sequence  $\{c_n\}$  by defining  $c_n = b_n$  for  $n = 1, \dots, 10^9$ , and  $c_n = a_n$  for all  $n > 10^9$ . Use the definition of the limit to prove that  $c_n \rightarrow a$ .

**6.** (20 points). Suppose  $f : A \rightarrow B$  and  $g : B \rightarrow A$  are both one-to-one and onto functions. Prove that the composition  $g \circ f : A \rightarrow A$  is one-to-one and onto.