Name ______________________________

Student ID # ______________________________

Math 1270-002
Fall 2009

EXAM I
Friday, September 25, 2009

Problem	Points	Score
1.	20	
2.	15	
3.	15	
4.	25	
5.	15	
6.	10	
	TOTAL	

(20 points) 1. Suppose the height in feet of a watermelon above the ground, at time t seconds, is described by $f(t) = -16t^2 + 256$.

(a) What is the *instantaneous velocity* of the watermelon at $t = 0$?

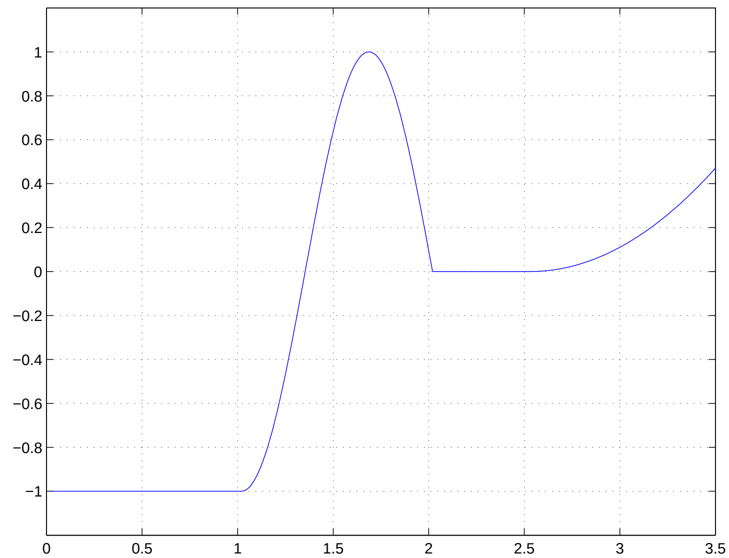
(b) What is the *average velocity* of the watermelon between $t = 0$ and the time it hits the ground?

(c) Find the secant line approximation to $f(t)$ between times $t = 1$ and $t = 3$.

(d) Find the time t at which the slope of the secant line from part (c) equals $f'(t)$.

(15 points) 2. Let $f(x) = \dfrac{1}{x-1}$. Use the *definition* of the derivative to find $f'(x)$.

(15 points) 3. Consider $f(x)$, whose graph looks like this:



(a) Sketch the graph of $f'(x)$.

(b) Based on the picture, is f differentiable in the interval $(0, 3.5)$?

(15 points) 4. Calculate the following.

(a) The tangent line to the graph $y = (x^2 - 1)(2x + 1)$ at the point $(1, 0)$.

(b) $\dfrac{dy}{dt}$, where $y(t) = \dfrac{x^2}{2 + \cos x}$

(c) $f'(x)$, where $f(x) = x(\cos x)(\sin x)$

(25 points) 5. (a) Does $\sin(1/x)$ have a limit as $x \to 0$? Why or why not?

(b) Does $x \sin(1/x)$ have a limit as $x \to 0$? Why or why not?

(c) If $\lim_{n\to\infty} a_n = 2$ and $\lim_{n\to\infty} b_n = 1$, find $\lim_{n\to\infty} 2(a_n^2 + 3b_n)$.

(d) Define $f(x) = \begin{cases} 1 - |x|, & \text{if } x \neq 0, \\ 0, & \text{if } x = 0. \end{cases}$ Find $\lim\limits_{x\to 0} f(x)$.
Is f continuous at $x = 0$?

(e) Suppose that given any number ϵ, you can find a $\delta > 0$ so that $|f(x) - 1| > \epsilon$ whenever $|x - 2| < \delta$. Does $\lim\limits_{x\to 2} f(x) = 1$?

(10 points) 6. At time $t = 0$, a large oil tanker is sailing in a fixed direction at velocity 2 m/s. It is accelerating at 1/2 m/s^2, in a vain attempt to outrun a small, fast pirate boat. The pirate boat is 900 m directly behind the tanker. The pirates, who studied calculus in pirate school, would like to travel at a constant speed v which will allow them to catch the tanker exactly at the time that the speeds of the two vessels match. What is the correct speed v, and at what time do the pirates catch the tanker?