Mathematics 1250-3 PRACTICE EXAM III Fall 2003

Remember to review homework exercises and the summaries after Chapters 4 and 5. Exam III covers through Section 6.1.

1. Let $f(x) = (x^2 - 3)e^x$, with $-\infty < x < \infty$.
   (a) Determine the intervals upon which f is increasing.
   (b) Determine the points at which f changes concavity.
   (c) Determine the point at which f attains its (global) minimum value.

2. Evaluate the following limits. If the limit does not exist, state why.
   (a) $\lim_{x\to\infty} \frac{e^x}{x^4}$.
   (b) $\lim_{x\to 1^+} \left(\frac{1}{x-1} - \frac{x}{\sqrt{x-1}} \right)$.
   (c) $\lim_{x\to e^+} (\ln x)^{1/(x-e)}$.

3. Recall that for a sphere of radius r, the surface area is $4\pi r^2$ and the volume is $\frac{4}{3}\pi r^3$. A spherical ball bearing is losing surface area due to wear at a rate of 10^{-3} mm^2 per day. When the ball bearing is 10 mm in radius, at what rate is it's volume changing?

4. Find the global extrema of the function $f(x) = \frac{x^2}{1+x}$ on the interval $[0, 2]$.

5. Calculate $\frac{d}{dx}\left(\int_0^{e^{2x}} \sin(t^2)\, dt \right)$.

6. Find the area of the bounded region between the curves $y = x^2$, and $x = y^2$.

7. Solve the initial value problem $y' = \frac{t^3}{y^2}$, $y(0) = 1$, $t \geq 0$.

8. Set up, but *do not evaluate*, an integral to calculate the volume of the solid of revolution obtained by revolving the bounded area between the curves $y = x + 1$ and $y = (x-1)^2$ around the x-axis.