Mathematics 1250-3 PRACTICE EXAM I Fall 2003

1. Calculate the following sums. If a particular sum is not finite, state this clearly and explain why.

   (a) $\displaystyle\sum_{j=2}^{\infty} \frac{1}{3^j}$ (b) $\displaystyle\sum_{k=3}^{5} k^2$ (c) $\displaystyle\sum_{n=1}^{\infty} \frac{1}{n}$

2. Calculate the following limits. If a particular limit does not exist, state this clearly and tell why.

   (a) $\displaystyle\lim_{x\to\sqrt{2}} 3x^2$ (b) $\displaystyle\lim_{x\to -1} \frac{x^2-x+2}{x+1}$ (c) $\displaystyle\lim_{x\to 0^+} \sqrt{x}\sin\left(\frac{1}{x^2}\right)$

   (d) $\displaystyle\lim_{x\to 2} f(x)$, where $f(x) = \begin{cases} x^3, & x \le 2 \\ x, & x > 2 \end{cases}$ (e) $\displaystyle\lim_{x\to 0} \sqrt[3]{\frac{8x^7+3x^5}{x^7+6x^2+2}}$

3. (a) Let $f(x) = \sqrt{x}$. Using the *definition* of the derivative, calculate $f'(x)$. Do the same for $g(x) = 1/x$.

   (b) Using your result from (a), find the equation of the line tangent to the graph of $f(x) = \sqrt{x}$ at $x = 1$. Do the same for $g(x) = 1/x$.

4. Let $f(x) = -x$ when $x \le 0, x \ne -1$; 2 when $x = -1$; $\sqrt{x}$ when $0 < x < 1$; $\sqrt[3]{3-x}$ when $x \ge 1$. Sketch the graph of $f(x)$.

   (a) For which points c does $\lim_{x\to c} f(x)$ exist? (b) For which points is f continuous?

   (c) For which points is f differentiable?

5. Let $f(x) = x + 2$ when $x \le 0$; $-\frac{1}{2}x + 2$ when $0 < x \le 2$; $\sqrt{x-2} + 1$ when $x > 2$. Sketch the graph of $f(x)$, and then using your result sketch the graph of $f'(x)$.

6. Find the derivative and antiderivative of $f(x) = 12x^5 + 5x^4 + x^2 + 2x + 1$

7. Calculate the following derivatives

   (a) $\displaystyle\frac{d}{dx}\left(\frac{x+1}{x-1}\right)$ (b) $\displaystyle\frac{d}{dx}(\sqrt{x})^3$ (c) $\displaystyle\frac{d}{dx}(x^3+3x^2+2x+1)^{-1}$

8. Let the position $x(t)$ of a particle at time t be given by $x(t) = 3t^2 - 2t + 1$. Find the instantaneous velocity $v(t)$ of the particle for any time t. Find the position of the particle when its velocity is zero.

9. On earth, the acceleration $a(t)$ due to gravity is essentially constant in time, with $a(t) = -g$, where $g = 9.8\ m/s^2$. On nearby planet Ψ, scientists have discovered how to vary their planet's gravitational force with time. If the acceleration due to gravity on Ψ is $a(t) = -t$, find find the position $x(t)$ of an object with initial velocity v_0 and initial position x_0. Using your expression for $x(t)$, find how long it will take for a ball thrown upward from the ground $x = 0$ on Ψ at $t = 0$ with initial velocity $6\ m/s$ to hit the ground.