1. Consider the matrix

\[ D_G = (d_{i,j}) = \begin{pmatrix} 
0 & 2 & 5 & 10 \\
3 & 0 & 4 & 5 \\
1 & 2 & 0 & 1 \\
2 & \infty & 3 & 0 \\
\end{pmatrix} \]

associated to a weighted directed graph \( G \) with 4 vertices \( v_1, \ldots, v_4 \). Here \( d_{i,j} \) is the length of the arrow from \( v_i \) to \( v_j \). Compute \( D_G \odot^3 G \) and determine the shortest path from \( v_1 \) to \( v_4 \).

2. Consider the tropical polynomial

\[ p(x) = 3x^3 \oplus 2x^2 \oplus 1x \oplus 2. \]

Draw the graph of \( y = p(x) \), and use it to find all roots and their multiplicities. Check that \( p(x) \) is a least coefficient polynomial, and write the tropical factorization of \( p(x) \) as stated in the Fundamental Theorem of Tropical Algebra. Then check, by multiplying out, that \( p(x) \) is indeed equal to the claimed factorization.

3. Draw the tropical curves defined by the following tropical polynomials and their dual complexes.
   (a) \( f(x, y) = 3x \oplus 1y \oplus (-2) \)
   (b) \( f(x, y) = 3x^2 \oplus xy \oplus 1y^2 \oplus 1x \oplus y \oplus 0 \)
   (c) \( f(x, y) = 4x^2 \oplus 1xy \oplus y^2 \oplus x \oplus y \oplus 1 \)
   (d) \( f(x, y) = 1x^2 \oplus xy \oplus y^2 \oplus 0 \)
   (e) \( f(x, y) = 6x^3 \oplus x^2y \oplus xy^2 \oplus 6y^3 \oplus 3x^2 \oplus (-1)xy \oplus 3y^2 \oplus 1x \oplus 1y \oplus 0 \)

4. Classify all combinatorial types of singular cubic curves of genus zero in the projective tropical plane with full support and no multiple edges reaching out to a line at infinity. For several combinatorial types: describe the dual Newton polygon, show a picture of the curve in the tropical projective plane, mark its singular points, and decide if it is irreducible or not.

5. Continuing from the previous exercise, for at least one combinatorial type find a tropical polynomial of degree 3 defining a cubic of that type.

6. Continuing from the previous two exercises, use a graph to describe how some of the various combinatorial types degenerate into each other.

7. Make a conjecture on how to recognize if a tropical curve without multiple edges is reducible from its dual complex.

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Possible topics for independent projects on tropical algebra and tropical curves:

- Tropical linear algebra.
- Classification and moduli space of tropical projective conics.
- Minkowski sum and reducibility of tropical curves.
- The group structure on a smooth tropical cubic curve.
- Intersection of tropical plane curves and tropical Bezout theorem.