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BOUNDARY HARNACK PRINCIPLE FOR SYMMETRIC STABLE PROCESSES

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Considerable progress has been made recently in studying the "fine properties" of symmetric stable processes. We are going to talk about these recent results. In particular, we are going to focus on the boundary Harnack principle for symmetric stable processes on any open set D: for nonnegative functions u and v in R^n, positive regular α-harmonic on $D \cap B(z, 2r)$, vanshing on $D^c \cap B(z, 2r)$, $z \in \partial D$, and $r > 0$, we have

$$C^{-1}\frac{u(x)}{v(x)} \leq \frac{u(y)}{v(y)} \leq C\frac{u(x)}{v(x)}, \qquad x, y \in B \cap B(z, 2r),$$

for some constant C depending on n, α and the ratio between r and the radius of the largest ball in $B \cap B(z, r)$. As applications of this boundary Harnack principle, we can show that for open sets with a certain interior fatness condition the Martin boundary can be identified with the Euclidean boundary.