

Stochastics Seminar

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Sample Path Properties of Anisotropic Gaussian Random Fields

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The class of anisotropic Gaussian random fields includes fractional Brownian sheets, certain operator-self-similar Gaussian random fields with stationary increments and the random string process. They arise naturally in many areas and can serve as more realistic models than fractional Brownian motion.

This talk is concerned with an (N, d) -fractional Brownian sheet $B^H = \{B^H(t), t \in \mathbf{R}_+^N\}$ with Hurst index $H = (H_1, \dots, H_N) \in (0, 1)^N$. Our interest lies in characterizing the anisotropic properties of B^H in terms of H .

First we prove that B^H has the property of *sectorial local non-determinism*. By using it as a technical tool, we study various sample path properties of B^H . In particular:

1. We determine the Hausdorff dimension of the image set $B^H(E)$ for an arbitrary Borel set $E \subseteq (0, \infty)^N$.
2. We provide sufficient conditions for $B^H(E)$ to be a Salem set or to have interior points.
3. We prove that if $\sum_{j=1}^N H_j^{-1} > d$ then B^H has a continuous local time process.