Errata to "Probability"

D. Khoshnevisan

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Many thanks are due to the following who have offered many comments, corrections, and suggestions: Simcha Barkai, Zoltán Buczolich, Matthew Daws, Richard Durrett, José García-Cuerva, Stacy Hill, Daniel Mauldin, Ron Reeder, ...

page 9: (updated on Sept 23, 2013) The line above (1.20) should be replaced with the following: "*non-empty* $A \subset \{1, \ldots, n\}$ *let*"

page 25: (updated on Dec 16, 2013) Lemma 3.29 should state that the domain of the said set function is a σ-algebra

page 37: (updated on Sept 23, 2013) Eq. (4.3) should read

$$\int (af + bg) \mathrm{d}\mu = a \int f \, \mathrm{d}\mu + b \int g \, \mathrm{d}\mu. \tag{4.3}$$

page 90: (updated on Sept 23, 2013) The last line of Note (15) is misleading, and should be corrected to the following: "*For more information on this topic, see the paper by Buczolich and Mauldin (1999).*"

page 93: (updated on Sept 23, 2013) In line -3, $N \geq 1$ has to be assumed to be sufficiently large.

page 102: (updated on Sept 23, 2013) Definition 7.19 should be written as follows: "*Define* $\mathbf{S}^{n-1} := \{x \in \mathbf{R}^n : \|x\| = 1\}$ *to be the* unit sphere *in* $\mathbf{R}^n$. *This is a metric space, once endowed with the usual Euclidean metric.*"

pages 103–104: (updated on Sept 23, 2013) The described proof of the uniqueness is flawed when $n > 1$. That is the material which begins with "Next we prove the more interesting uniqueness portion," on ¶2, and concludes at the end of the proof. Here is a correct proof (due to J. P. R. Christensen (1970), *Math. Scand.* **26**, 103–106): "*Let* μ *and* ν *be two uniform probability measures on* $\mathscr{B}(\mathbf{S}^{n-1})$. *If* $x \in \mathbf{S}^{n-1}$ *and* $r > 0$, *then we write* $B(x, r)$ *for the open ball of radius* r *about* x. *Note that* $\bar{\mu}(r) := \mu(B(x, r))$ *and* $\bar{\nu}(r) := \nu(B(x, r))$ *are positive and do not depend on* x. *Also note that if* U *is a nonvoid open set in* $\mathbf{S}^{n-1}$, *then*

$$\lim_{r \to 0} \frac{\mu(U \cap B(x, r))}{\bar{\mu}(r)} = \lim_{r \to 0} \frac{\nu(U \cap B(x, r))}{\bar{\nu}(r)} = 1 \qquad {}^{\forall} x \in U.$$

Therefore, by the dominated convergence theorem (p. 46),

$$\mu(U) = \lim_{r\to 0} \frac{1}{\bar{\nu}(r)} \int_U \nu(U \cap B(x\,, r))\, \mu(\mathrm{d}x).$$

An application of the Fubini–Tonelli theorem (p. 55) reveals that

$$\begin{aligned}\mu(U) &= \lim_{r\to 0} \frac{1}{\bar{\nu}(r)} \int_U \mu(U \cap B(x\,, r))\, \nu(\mathrm{d}x) \\ &= \lim_{r\to 0} \frac{\bar{\mu}(r)}{\bar{\nu}(r)} \int_U \frac{\mu(U \cap B(x\,, r))}{\bar{\mu}(r)}\, \nu(\mathrm{d}x) \\ &= \lim_{r\to 0} \frac{\bar{\mu}(r)}{\bar{\nu}(r)} \cdot \nu(U),\end{aligned}$$

owing to a second application of the dominated convergence theorem. Since the roles of μ and ν are interchangeable, it follows that $\mu(U) = \nu(U)$ for all nonvoid open sets U, whence $\mu = \nu$."

page 106: (updated on Sept 23, 2013)

- Line 3: $g(x\sqrt{n})$ should be $g(x/\sqrt{n})$
- Line −6: $8(|a|^3 + |b|^3)$ can be improved to $4(|a|^3 + |b|^3)$

page 107: (updated on Sept 23, 2013)

- Line 4: $S_n - S'_n = \sum_{i=1}^n \mathbf{1}_{\{|X_i|>\epsilon\sqrt{n}\}}$ [$>$ in place of $\geq$]
- Eq. (7.76) is missing $\sup_z |g'(z)|$ on the right-hand side

page 112: (updated on Nov 6, 2013) It should say also that X and Y have the same variance.

page 122: (updated on Sept 23, 2013)

- Line 2 of (8.3): b_1 should be replaced by a_1
- One line before *Step 4*: ξ should be written as $\xi^+ - \xi^-$

page 128: (updated on Sept 24, 2013)

- Remark 8.19 should contain the additional proviso that $p \in [1\,, \infty)$
- In Doob's decomposition: Every semimartingale is a *sum* of a sub- and a supermartingale

page 133: (updated on Sept 24, 2013) The denominator in the second formula in (8.24) should be $\zeta^{h+g} - \zeta^g$ [the $p \neq 1/2$ case]

page 149: (updated on Nov 27, 2013) One line above (8.89), it should state "$x \in \mathscr{F}_n^0$" and not "$\omega \in \mathscr{F}_0^n$." One line above this, it should say "$J \in \mathscr{F}_n^0$" and not "$J \in \mathscr{F}_0^n$."

page 154: (updated on Sept 24, 2013) In (8.111), little-o should be replaced by big-O

page 174: (updated on Sept 23, 2013)

- Line 1 of (9.45), line 1 of (9.46), and line 1 of (9.47) should all say "$W(T+t_i)-W(T)$" in place of $W(T+t_i)-W(t_i)$"
- One line above *Step 2.* $A=\Omega$ [and not $A=\mathbf{R}$]