Math 6070-1, Spring 2014; Assignment #1

Assigned on: Friday January 17, 2014
Due: Wednesday January 22, 2014

1. Complete reading the Probability Primer at http://www.math.utah.edu/~davar/math6070/2014/Probability.pdf.

2. Let X denote a random variable with a socalled "double exponential" density function,

$$f(x) = \tfrac{1}{2}\mathrm{e}^{-|x|} \qquad (-\infty < x < \infty).$$

   (a) Compute the moment generating function and the characteristic function of X.

   (b) Use your computations to evaluate $\mathrm{E}(X^n)$ for every integer $n \geq 1$. Justify your method.

3. Prove that if $X_1, X_2, \ldots, X_n$ form an i.i.d. sample from a Uniform$(0\,,1)$ distribution, then $\prod_{j=1}^{n} X_j^{1/n}$ converges to $1/\mathrm{e}$ in probability as $n \to \infty$.

4. Suppose $X_1, X_2, \ldots$ are i.i.d. random variables, selected from a Poisson(1) distribution. Define $S_n := X_1 + \cdots + X_n$ for every $n \geq 1$

   (a) Compute the distribution of S_n for every $n \geq 1$.

   (b) Use the central limit theorem to approximate $\mathrm{P}\{S_{100} \leq 120\}$.