4.6. First, let us prove this for $k = 1$: For all closed intervals $[a,b]$, we can find continuous functions $f_n \downarrow \mathbf{1}_{[a,b]}$. By the monotone convergence theorem, $\mu([a,b]) = \nu([a,b])$. Therefore, μ and ν agree on the algebra generated by closed intervals. Because this algebra genarates the Borel σ-algebra $\mathscr{B}(\mathbf{R})$, the Carathéodory's extension theorem prove that $\mu = \nu$.

In order to carry this program out when $k > 1$, we approximate the function $\mathbf{1}_{[a_1,b_1]\times\cdots\times[a_k,b_k]}(x_1,\ldots,x_k) = \mathbf{1}_{[a_1,b_1]}(x_1) \times \cdots \times \mathbf{1}_{[a_k,b_k]}(x_k)$ by continuous functions of the form $f_n^1(x_1) \times \cdots \times f_n^k(x_k)$. Then proceed as in the $k = 1$ case, using the fact that hyper-cubes of the form $[a_1,b_1] \times \cdots \times [a_k,b_k]$ generate $\mathscr{B}(\mathbf{R}^k)$.