4.27. If f is continuously differentiable with compact support, then as the hint suggests, we can integrate by parts to find that $\int_{-\infty}^{\infty} xf(x)f'(x)\,dx = -\frac{1}{2}\int_{-\infty}^{\infty} f^2(x)\,dx$. [If $u := x$, and $v' := ff'$ then $u' = 1$ and $v = \frac{1}{2}f^2$. Because of this and the compact-support property of f, $\int uv' = -\frac{1}{2}\int f^2$.] Apply the Cauchy–Schwarz inequality to find that $\frac{1}{2}\int_{-\infty}^{\infty} f^2 \le (\int_{-\infty}^{\infty} x^2 f^2)^{1/2}(\int_{-\infty}^{\infty}(f')^2)^{1/2}$, as planned. Note that we have not used the C^1 property of f; only that f' exists a.e. Therefore, the same bound is valid for all a.e.-differentiable functions f of compact support. Suppose next that f' exists a.e. but f does not have compact support. Replace f by $f\mathbf{1}_{[-n,n]}$ to deduce that $\frac{1}{2}\int_{-n}^{n} f^2 \le (\int_{-n}^{n} x^2 f^2)^{1/2}(\int_{-n}^{n}(f')^2)^{1/2}$. Let $n \uparrow \infty$ and appeal to the monotone convergence theorem to obtain the general result.