

4.12. Apply Fatou's lemma to $Y_k := \sup_n |X_n| - X_k$ to find that $E[\liminf_k Y_k] \leq \liminf_k E[Y_k]$. Solve to find that $\limsup E X_n \leq E \limsup X_n$. For a counter-example consider X_n which is equal to n with probability $1/n$ and 0 with probability $1 - (1/n)$. Evidently, $\limsup_{n \rightarrow \infty} X_n = \infty$, yet $E X_n = 1$.