

Math 6010
Solutions to homework 1

- 2, p. 15. (a) Notice that $X_1 - 2X_2 + X_3 = \mathbf{a}'\mathbf{X}$, where $\mathbf{a} := (1, -2, 1)'$. Therefore,
 $\text{Var}(X_1 - 2X_2 + X_3) = \mathbf{a}'\text{Var}(\mathbf{X})\mathbf{a}$, which is

$$(1, -2, 1) \begin{pmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix} = (1, -2, 1) \begin{pmatrix} 4 \\ -4 \\ 6 \end{pmatrix} = 18.$$

- (b) Write $\mathbf{Y} = \mathbf{A}\mathbf{X}$, where

$$\mathbf{A} := \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}.$$

Therefore, $\text{Var}(\mathbf{Y}) = \mathbf{A}\text{Var}(\mathbf{X})\mathbf{A}'$; that is,

$$\text{Var}(\mathbf{Y}) = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} 5 & 2 & 3 \\ 2 & 3 & 0 \\ 3 & 0 & 3 \end{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 12 & 15 \\ 15 & 21 \end{pmatrix}.$$