Math 5090–001, Fall 2009
Solutions to Assignment 8

Chapter 14, Problem 1. (a) $H_0:\ m = 0.5$ versus $H_a:\ m > 0.5$.

Let T denote the number of data points above 0.5. Then, $T \sim \text{BIN}(20\,,1/2)$, provided that H_0 is true. Let us compute the P-value:

In the sample, the observed value of our T statistic is 10. Therefore,

$$P\text{-value} = \mathrm{P}\left\{T \geq 10 \,\middle|\, H_0\right\} = \mathrm{P}\left\{\text{BIN}(20\,,1/2) \geq 10\right\} > \frac{1}{2}.$$

Since P-value is $\gg 0.05$ we do not reject.

(b) $H_0:\ m = 0.25$ versus $H_a:\ m > 0.25$.

Let T denote the number of data points above 0.25. Then, $T \sim \text{BIN}(20\,,1/2)$, provided that H_0 is true. Let us compute the P-value:

In the sample, the observed value of our T statistic is 18. Therefore,

$$\begin{aligned} P\text{-value} &= \mathrm{P}\left\{T \geq 18 \,\middle|\, H_0\right\} = \mathrm{P}\left\{\text{BIN}(20\,,1/2) \geq 18\right\} \\ &= 1 - 0.9998 = 0.0002. \end{aligned}$$

Since P-value is $\ll 0.1$ we reject.

Chapter 14, Problem 4. Let $Q(0.1)$ denote the 10th percentile. We want to test

$$H_0:\ Q(0.1) = 16000 \qquad H_a:\ Q(0.1) > 16000.$$

Let T denote the number of data points above 16000. Then, $T \sim \text{BIN}(20\,,0.9)$, provided that H_0 is true. Let us compute the P-value:

In the sample, the observed value of our T statistic is 15. Therefore,

$$\begin{aligned} P\text{-value} &= \mathrm{P}\left\{T \geq 15 \,\middle|\, H_0\right\} = \mathrm{P}\left\{\mathrm{BIN}(20\,,0.9) \geq 15\right\} \\ &= 1 - 0.0113 = 0.9887. \end{aligned}$$

Don't reject.

Chapter 14, Problem 7(b). The sample size is $n = 60$, and the random interval $(X_{i:n}\,, X_{j:n})$ is a 95% CI for the median weight, provided that i and j are chosen so that

$$\mathrm{BIN}(j-1\,;60\,,1/2) - \mathrm{BIN}(i-1\,;60\,;1/2) \approx 0.95.$$

By the CLT, the BIN(60 , 1/2) is basically N(30 , 15). Therefore, we want

$$\begin{aligned} 95\% &\approx \mathrm{BIN}(j-1\,;60\,,1/2) - \mathrm{BIN}(i-1\,;60\,;1/2) \\ &\approx \Phi\left(\frac{j-1-30}{\sqrt{15}}\right) - \Phi\left(\frac{i-1-30}{\sqrt{15}}\right). \end{aligned}$$

Might as well choose i and j to have symmetric intervals; i.e.,

$$\frac{j-1-30}{\sqrt{15}} = -\frac{i-1-30}{\sqrt{15}}. \tag{1}$$

In that case, we want

$$97.5\% \approx \Phi\left(\frac{j-31}{\sqrt{15}}\right);$$

i.e.,

$$1.96 \approx \frac{j-31}{\sqrt{15}} \quad \Rightarrow \quad j \approx 31 + 1.96\sqrt{15} \approx 38.591047358566537.$$

Therefore, we can take $j = 39$ [why not 38?]. And i is computed from (1) as follows: $8 = j - 31 = -(i - 31)$; i.e., $i = 31 - 8 = 23$. Thus, an approximate [conservative-ish] CI for the median weight is $(X_{23:60}\,, X_{39:60}) = (5.17\,, 5.24)$.

Chapter 14, Problem 12. **[not graded]** Let $p :=$ the true proportion who prefers B. We want $H_0 : \ p = \frac{1}{2}$ versus $H_s : \ p < \frac{1}{2}$. Do a binomial test from the previous chapters.