**Math 5090–001, Fall 2009**

**Solutions to Assignment 7**

**Chapter 13, Problem 1.** You can use a $\chi^2$ test; but a normal approximation works slightly better here [the end results are more or less the same in both cases.] We have $n = 100$ and $\hat{p} = 0.2$. The test statistic is

$$S := \frac{\hat{p} - p_0}{\sqrt{p_0(1-p_0)/n}} = \frac{0.2 - 0.3}{\sqrt{0.3 \times 0.7/100}} \approx -2.182179.$$

The distribution of $S$ is approximately $\mathrm{N}(0\,,1)$ by the CLT. Therefore, we find our 2-sided $P$-value to be 0.0292. Certainly reject $H_0$ at 5%.

If we wanted a one-sided test, then the answer would depend on how we set up our hypotheses. But the natural one is $H_0: \;p = 0.3$ versus $H_a: \;p < 0.3$. In that case, the $P$-value is 0.0146. Again, reject strongly at 5%. [The $\chi^2$ test will miss this part!]

**Chapter 13, Problem 6.** We compute the $\chi^2$ table:

| | spades | hearts | diamonds | clubs |
|---|---|---|---|---|
| observed | 6 | 8 | 9 | 13 |
| expected | 9 | 9 | 9 | 9 |

This leads to a $\chi^2$ statistic:

$$\chi^2 := \frac{(6-9)^2}{9} + \frac{(8-9)^2}{9} + \frac{(9-9)^2}{9} + \frac{(13-9)^2}{9} \approx 2.9.$$

Therefore, we can use a $\chi^2_3$ table to find that $P$-value $\approx 0.4$. Therefore, we do not reject at just about every reasonable level.

**Chapter 13, Problem 10.** The observed frequency table is:

| | Mech. | Elect. | Other |
|---|---|---|---|
| Design 1 | 50 | 30 | 60 |
| Design 2 | 40 | 30 | 40 |

The observed relative frequencies (as probabilities, out of a total of 250 trials) are:

| | Mech. | Elect. | Other | |
|---|---|---|---|---|
| Design 1 | 0.2 | 0.12 | 0.24 | 0.56 |
| Design 2 | 0.16 | 0.12 | 0.16 | 0.44 |
| | 0.36 | 0.24 | 0.4 | |

If "$H_0$ : independent" is true, then our estimate for the joint probabilities should be—based on the preceding marginals— obtained by multiplying into the table as follows:

| | Mech. | Elect. | Other | |
|---|---|---|---|---|
| Design 1 | 0.2016 | 0.1344 | 0.224 | 0.56 |
| Design 2 | 0.1584 | 0.1056 | 0.176 | 0.44 |
| | 0.36 | 0.24 | 0.4 | |

[For instance, the $(1\,,1)$ is obtained as $0.56 \times 0.36 \approx 0.2016$.] Turn the preceding into a table of expected frequencies [after estimation] by multiplying everything by 250:

| | Mech. | Elect. | Other |
|---|---|---|---|
| Design 1 | 50.4 | 33.6 | 56 |
| Design 2 | 39.6 | 26.4 | 44 |

Therefore,

$$\chi^2 = \frac{(50-50.4)^2}{50.4} + \frac{(30-33.6)^2}{33.6} + \frac{(60-56)^2}{65} + \frac{(40-39.6)^2}{39.6} + \frac{(30-26.4)^2}{26.4} + \frac{(40-44)^2}{44}$$
$$\approx 1.533.$$

We use a $\chi^2_{(\text{row}-1)(\text{col}-1)} = \chi^2_2$ table in order to find that $P$-value $\approx 0.7$. Therefore, "do not reject."

**Chapter 13, Problem 14.** (a) First of all, under $H_0$, the pdf is $f(x) = 0.01\exp(-0.01x)$ for $x \geq 0$. Therefore, $\mathrm{P}\{a \leq X < b\} = \mathrm{e}^{-a} - \mathrm{e}^{-b}$ whenever $0 < a < b$.

I will test to see if the correct proportion fall in the 5 intervals $[0\,,20]$, $(20\,,50]$, $(50\,,80]$, $(80\,,100]$, and $(100\,,\infty)$. [We can't have too many more intervals, since the data size is only 40.]

| | observed | expected |
|---|---|---|
| $0 \leq x < 20$ | 8 | 7.25 |
| $20 \leq x < 50$ | 7 | 8.48 |
| $50 \leq x < 80$ | 9 | 6.28 |
| $80 \leq x < 100$ | 4 | 3.25 |
| $x \geq 100$ | 12 | 14.7 |

Therefore,

$$\chi^2 = \frac{(8-7.25)^2}{7.25} + \cdots + \frac{(12-14.7)^2}{14.7} \approx 2.$$

Based on a $\chi^2_4$, we have $P$-value $\approx 0.7$. No reason to reject.

(b) If $X = \text{EXP}(\mu)$ for some $\mu > 0$, then $\text{E}X = 1/\mu$. Therefore, we first estimate $\text{E}X$ by $\bar{X}_{50} \approx 93.15$. Thus, we can test for $H_0:\ \text{EXP}(0.0107)$.

| | observed | expected |
|---|---|---|
| $0 \leq x < 20$ | 8 | 7.73 |
| $20 \leq x < 50$ | 7 | 8.89 |
| $50 \leq x < 80$ | 9 | 6.44 |
| $80 \leq x < 100$ | 4 | 3.27 |
| $x \geq 100$ | 12 | 13.67 |

Therefore,

$$\chi^2 = \frac{(8-7.73)^2}{7.73} + \cdots + \frac{(12-13.67)^2}{13.67} \approx 1.8.$$

Based on a $\chi^2_4$, we have $P$-value $\approx 0.78$. No reason to reject. [The $P$-value for (b) should be larger than the $P$-value for (a). Why?]