Math 5090–001, Fall 2009
Solutions to Assignment 6

Chapter 12, Problem 38. We have, for a Poiss(μ), $H_0: \ \mu = 1$ versus $H_a: \ \mu = 2$. In general,

$$f(x\,;\mu) = \frac{\mathrm{e}^{-\mu}\mu^x}{x!} \qquad \text{for } x = 0, 1, \ldots .$$

(a) The SPRT depends on likelihood ratios:

$$\lambda_m = \frac{f(X_1\,;1)\cdots f(X_m\,;1)}{f(X_1\,;2)\cdots f(X_m\,;2)} = \mathrm{e}^m 2^{-S_m},$$

where $S_m := X_1 + \cdots + X_m$. Therefore, if $\bar{X}_m := S_m/m$, then

$$k_0 < \lambda_m < k_1 \qquad \text{if and only if} \qquad -\log_2(\mathrm{e}k_1) < \bar{X}_m < -\log_2(\mathrm{e}k_0),$$

where "$\log_2$" denotes logarithm in base 2. Now proceed as in SPRT.

(b) We know from §12.10 that a reasonable estimate is

$$\mathrm{E}(N\,|\,H_a) \approx \frac{(1-\beta)\ln[\alpha/(1-\beta)] + \beta\ln[(1-\alpha)/\beta]}{\mathrm{E}(Z\,|\,H_a)} = \frac{0.08\ln(1/8) + 0.02\ln(9/2)}{\mathrm{E}(Z\,|\,H_a)};$$

it suffices to compute $\mathrm{E}(Z\,|\,H_a)$. Recall that

$$Z = \ln f(X\,;1) - \ln f(X\,;2) = 1 - X\ln 2.$$

Therefore, $\mathrm{E}(Z\,|\,H_a) = 1 - 2\ln 2 \approx -0.3862944$, and hence $\mathrm{E}(N\,|\,H_a) \approx 11.4297$, which ought to be interpreted conservatively as 12.